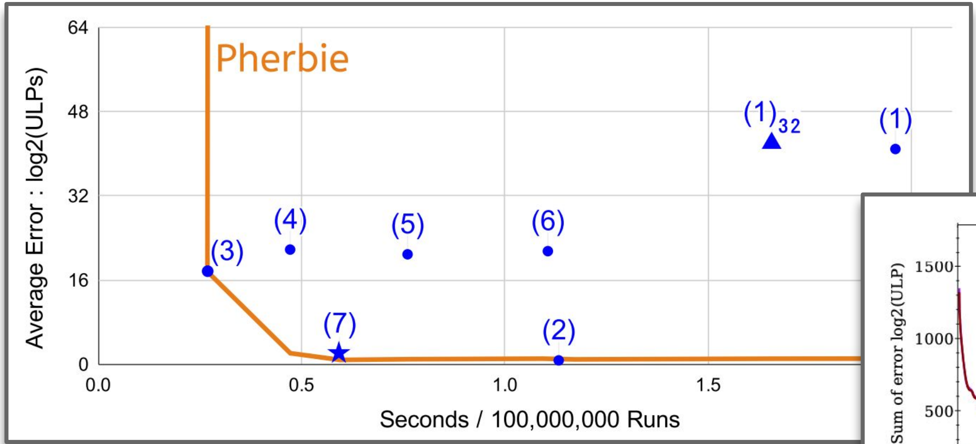
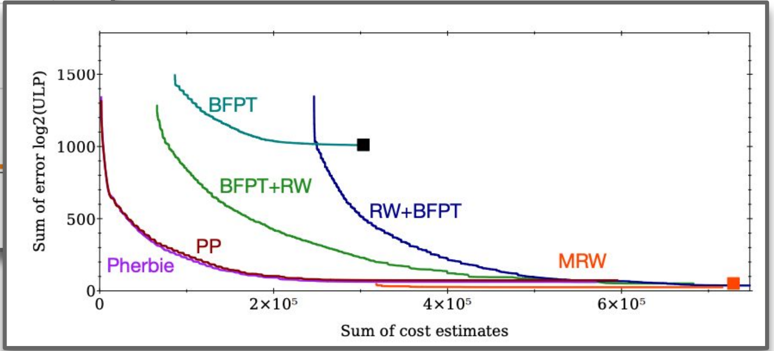
Combining Precision Tuning and Rewriting

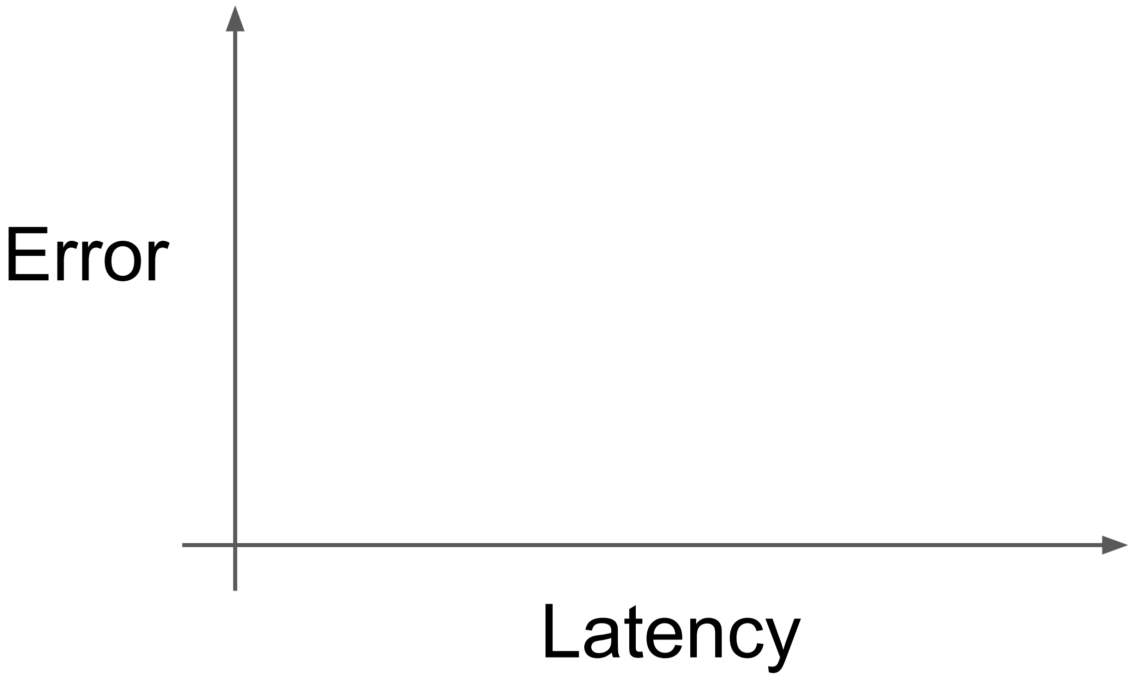


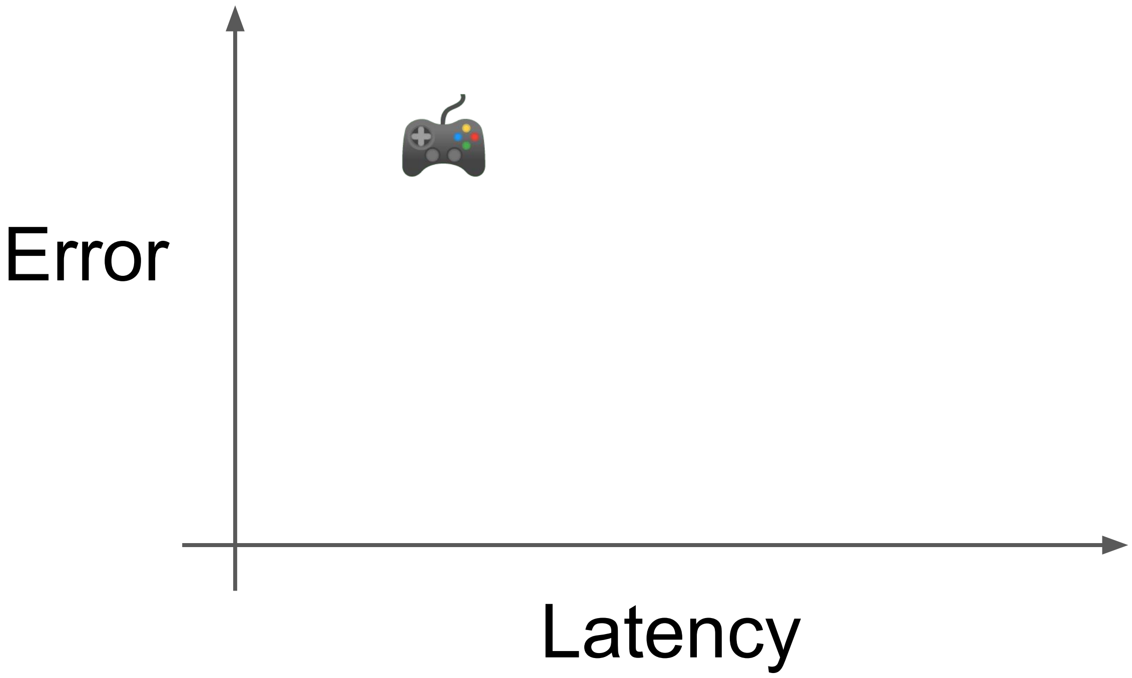


Brett Saiki, Oliver Flatt,

Chandrakana Nandi, Pavel Panchekha, Zachary Tatlock

HERBIE


Error
Latency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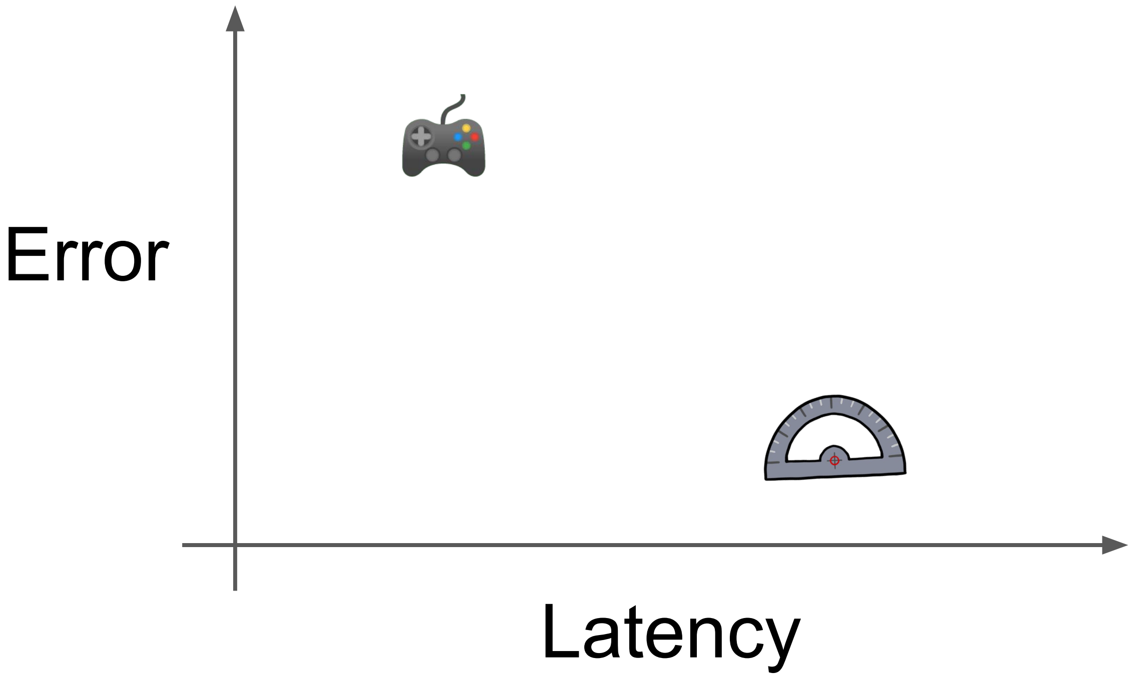
Error
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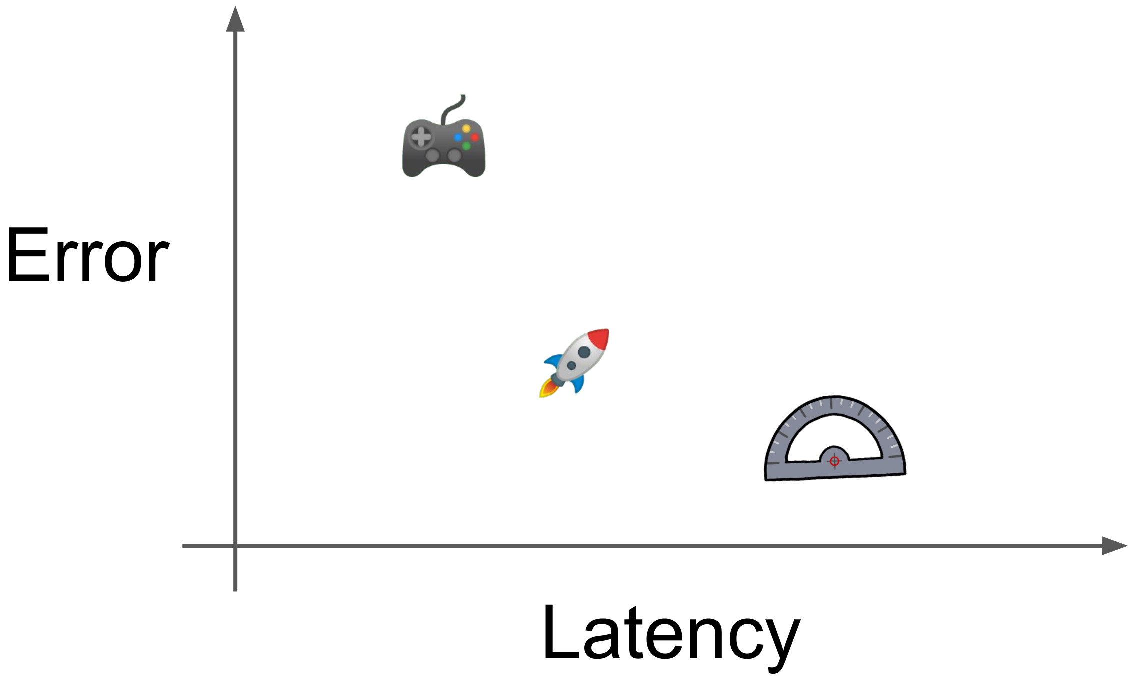

Error
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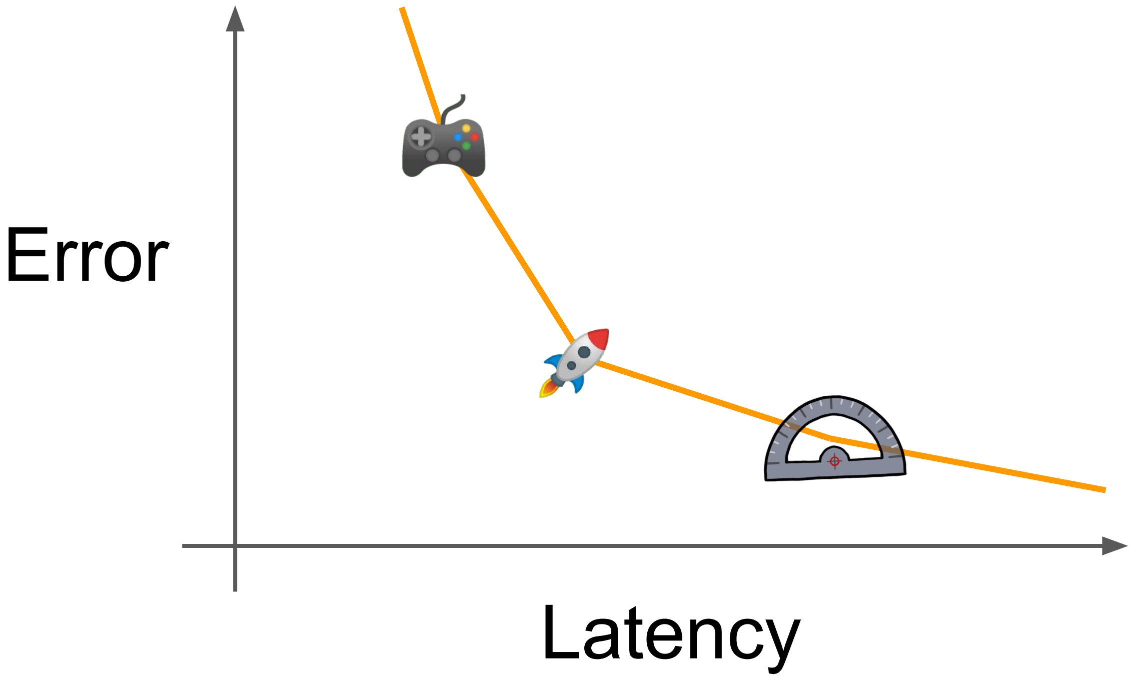
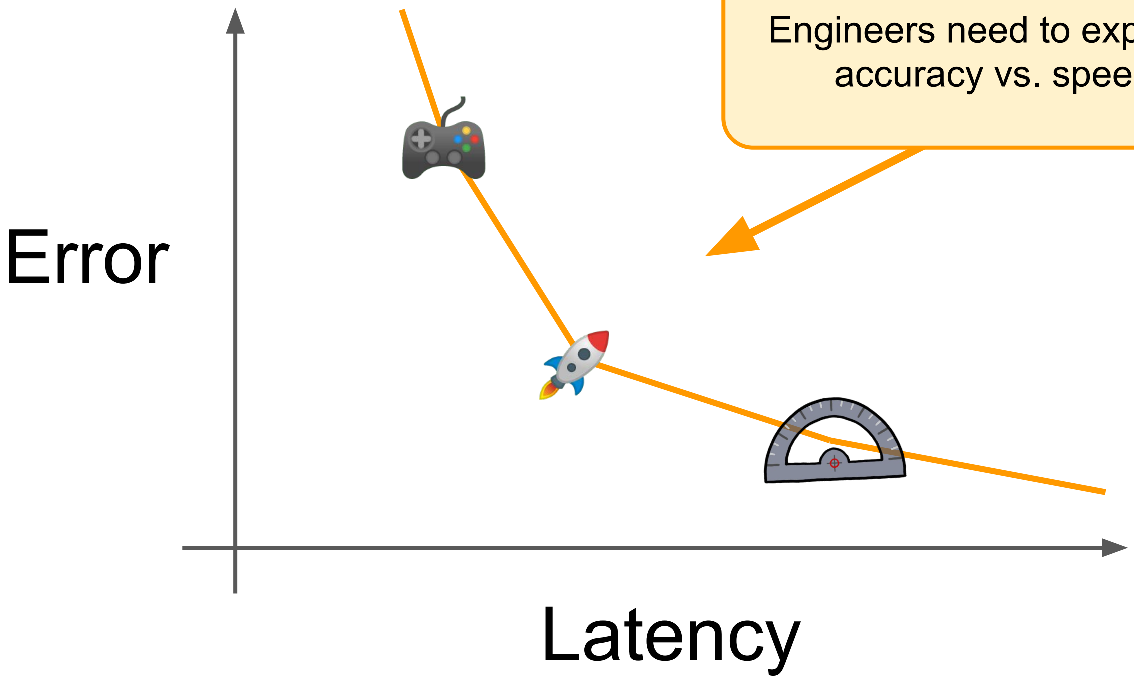

Error
Latency

No one tradeoff is right for every application.

Engineers need to explore the Pareto frontier of optimal accuracy vs. speed candidate implementations!



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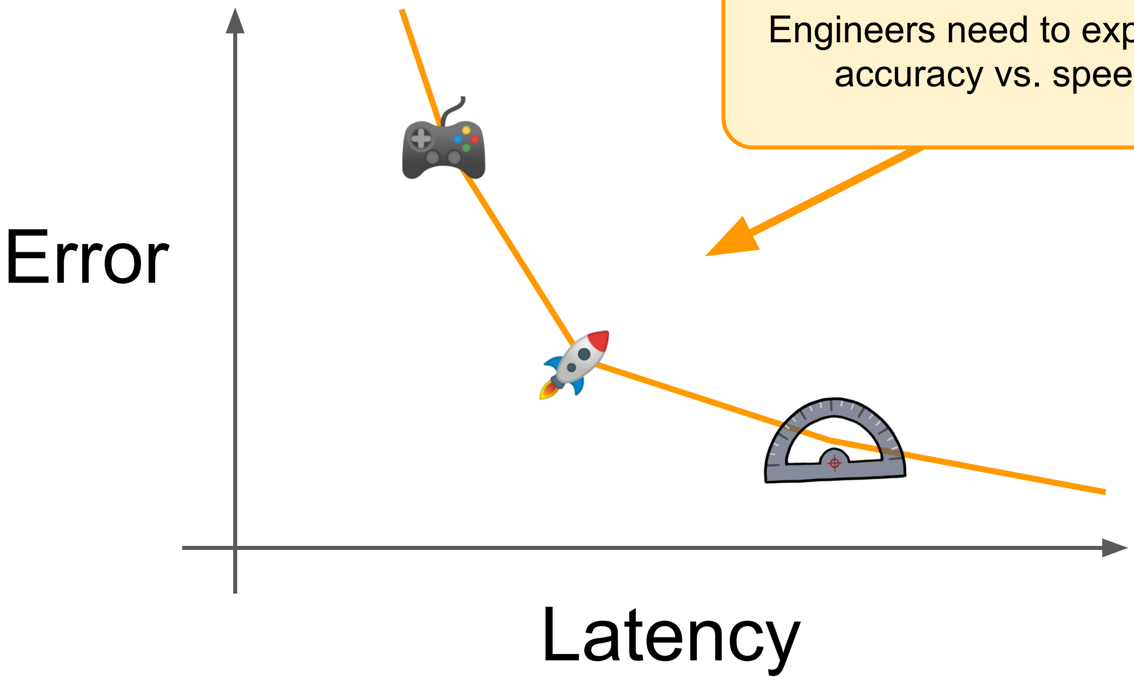

Precision Tuning

e.g., lower 64-bit ⇒ 32-bit

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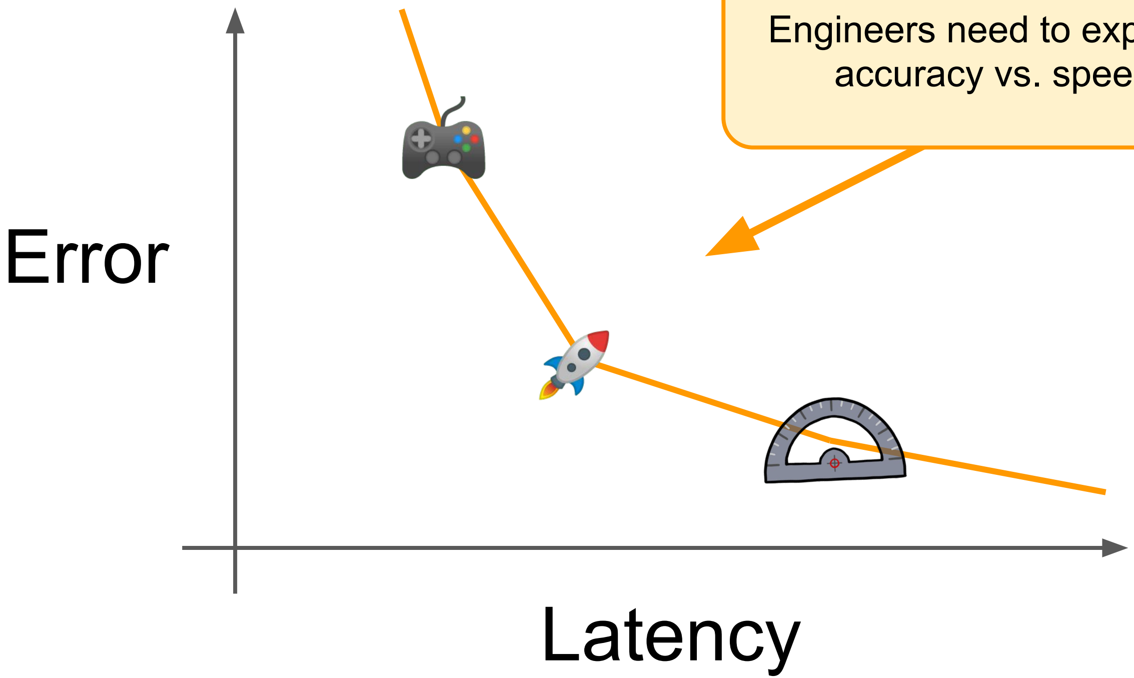

Precision Tuning

e.g., lower 64-bit ⇒ 32-bit

Program Rewriting

e.g., take series expansion

Precision Tuning

Program Rewriting

Precision Tuning

Program Rewriting

Lower bitwidth ⇒ higher throughput

- Major barrier: the memory wall!
- Enable more vectorization, etc.

Difficult to tell where lowering is safe

- Accums. large, but elts small?
- Past work adapts *delta debugging*
    - [Khalifa et al. FTSCS '19]
    - [Rubio-González et al. SC '13]

Precision Tuning

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Program Rewriting

Avoid pitfalls and/or use coarser approx

- Avoid cancellation, intro series
- e.g., generally want `(x + 1) - x ⇒ 1`

Difficult to find / carry out good rewrites

- Need to guide rewrite search
- Past work applies PL *synthesis*
    - [Schkufza et al. PLDI '14]
    - [Panchekha et al. PLDI '15]

How to optimize?

$$\sqrt{\frac{e^{2x} - 1}{e^x - 1}}$$

How to optimize $\sqrt{\frac{e^{2x} - 1}{e^x - 1}}$ via precision tuning?

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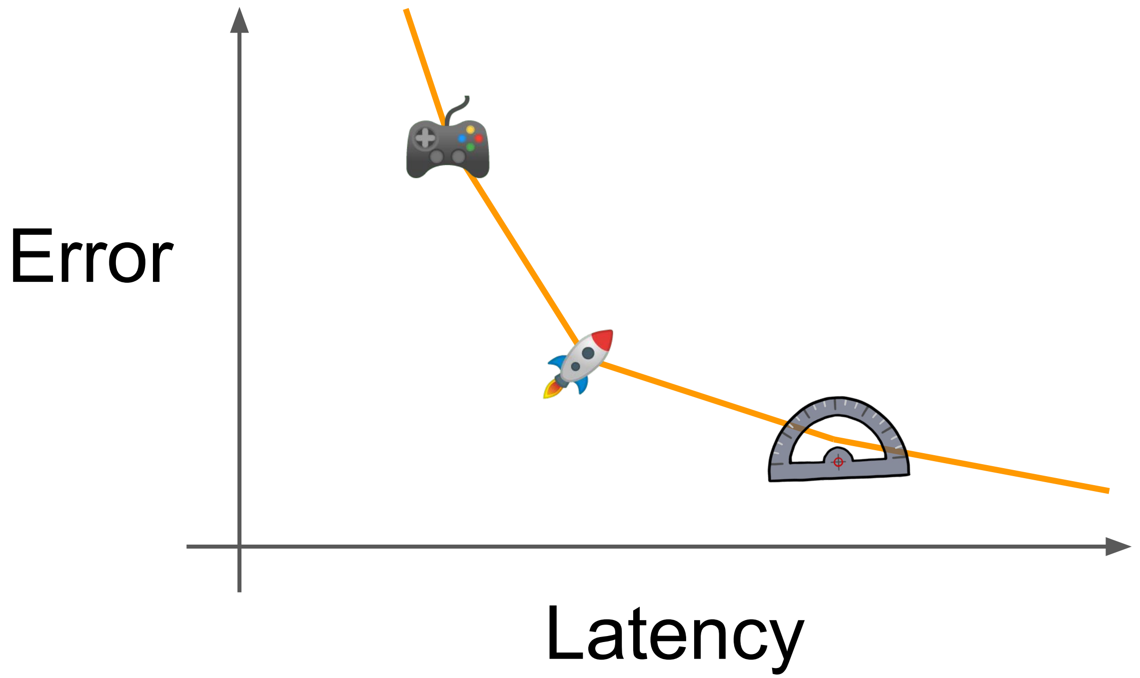
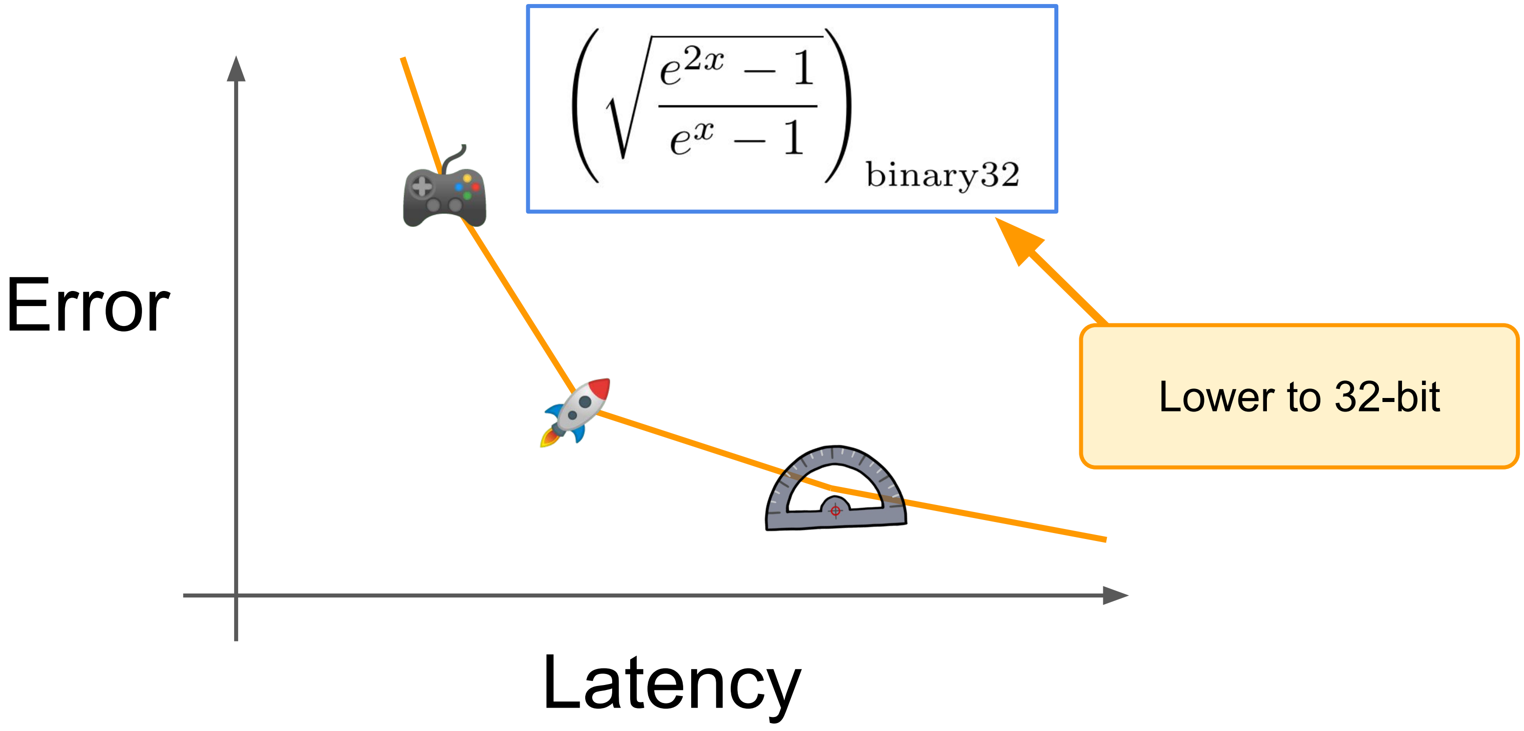


How to optimize $\sqrt{\frac{e^{2x}-1}{e^{x}-1}}$ via precision tuning?



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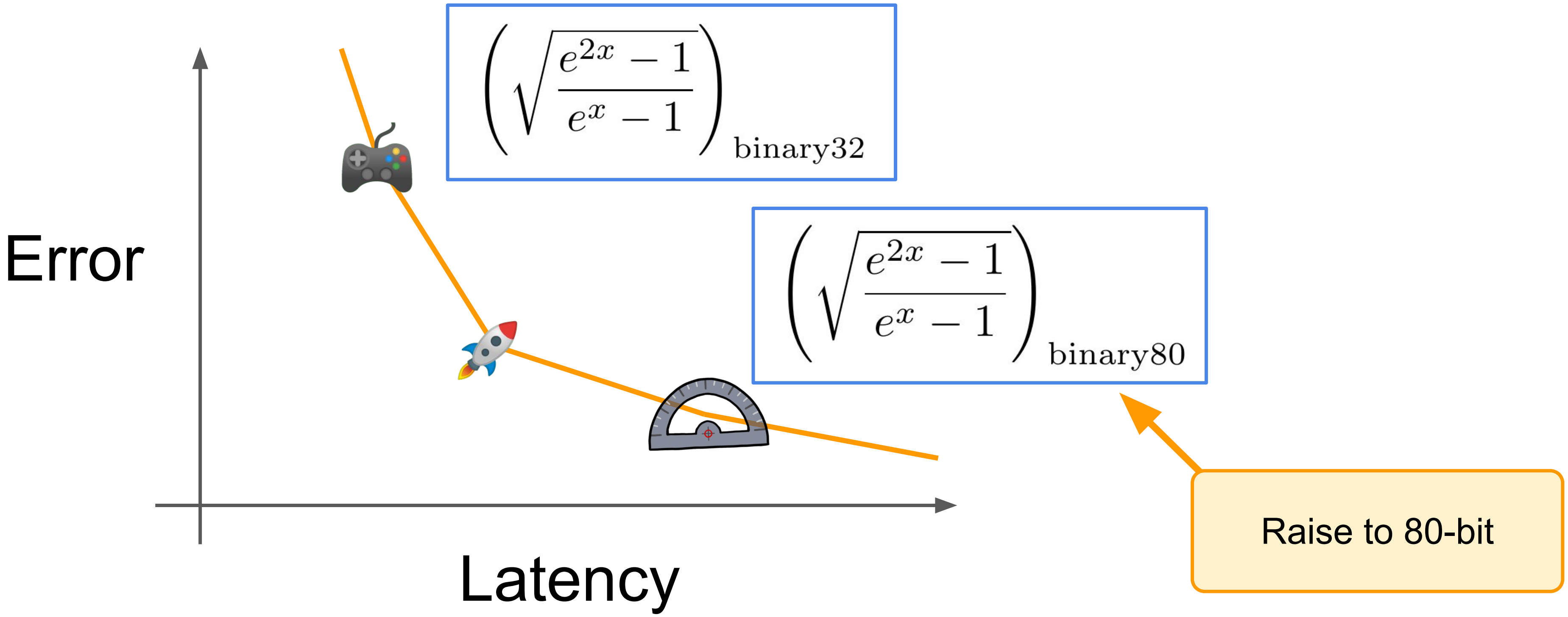

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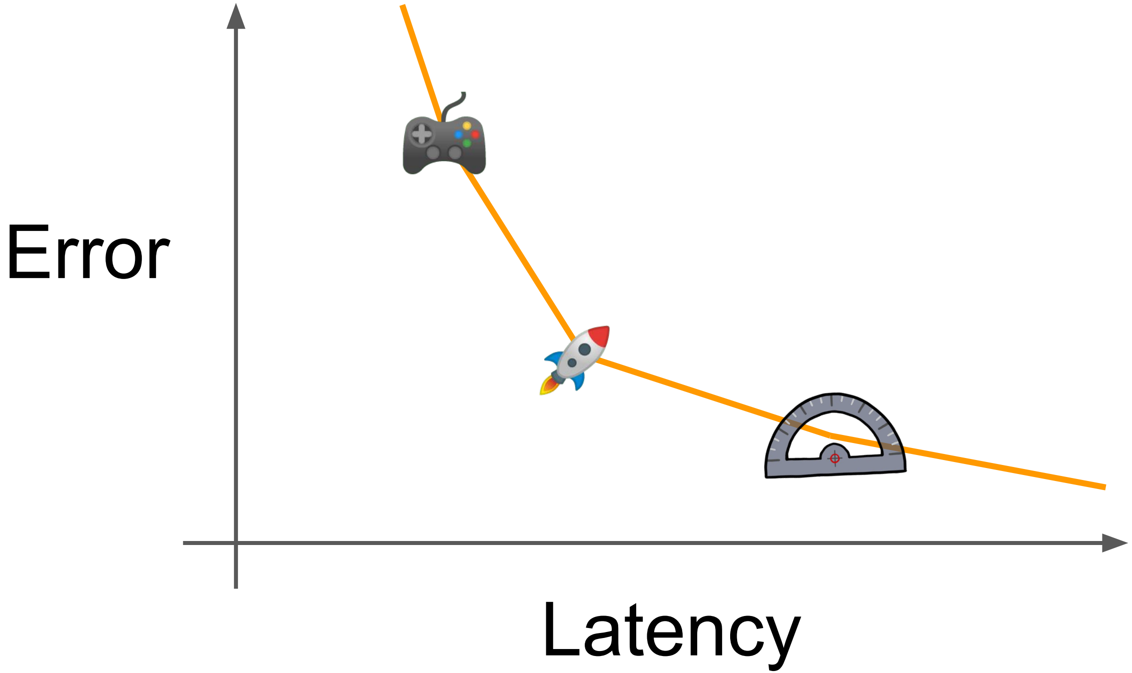

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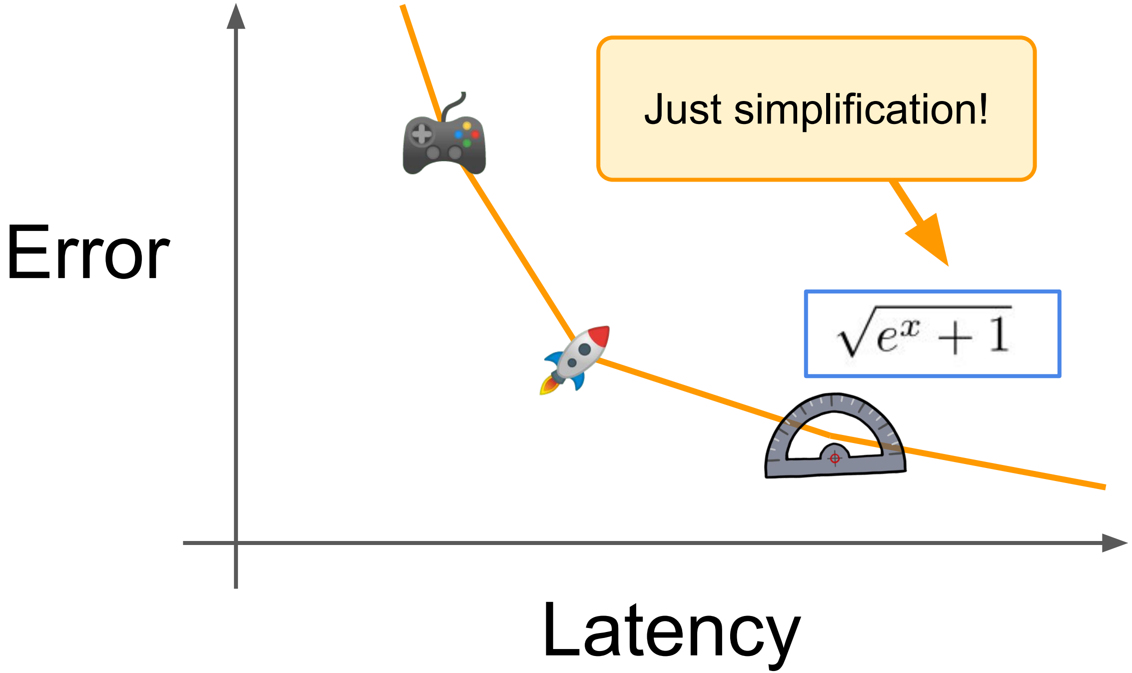

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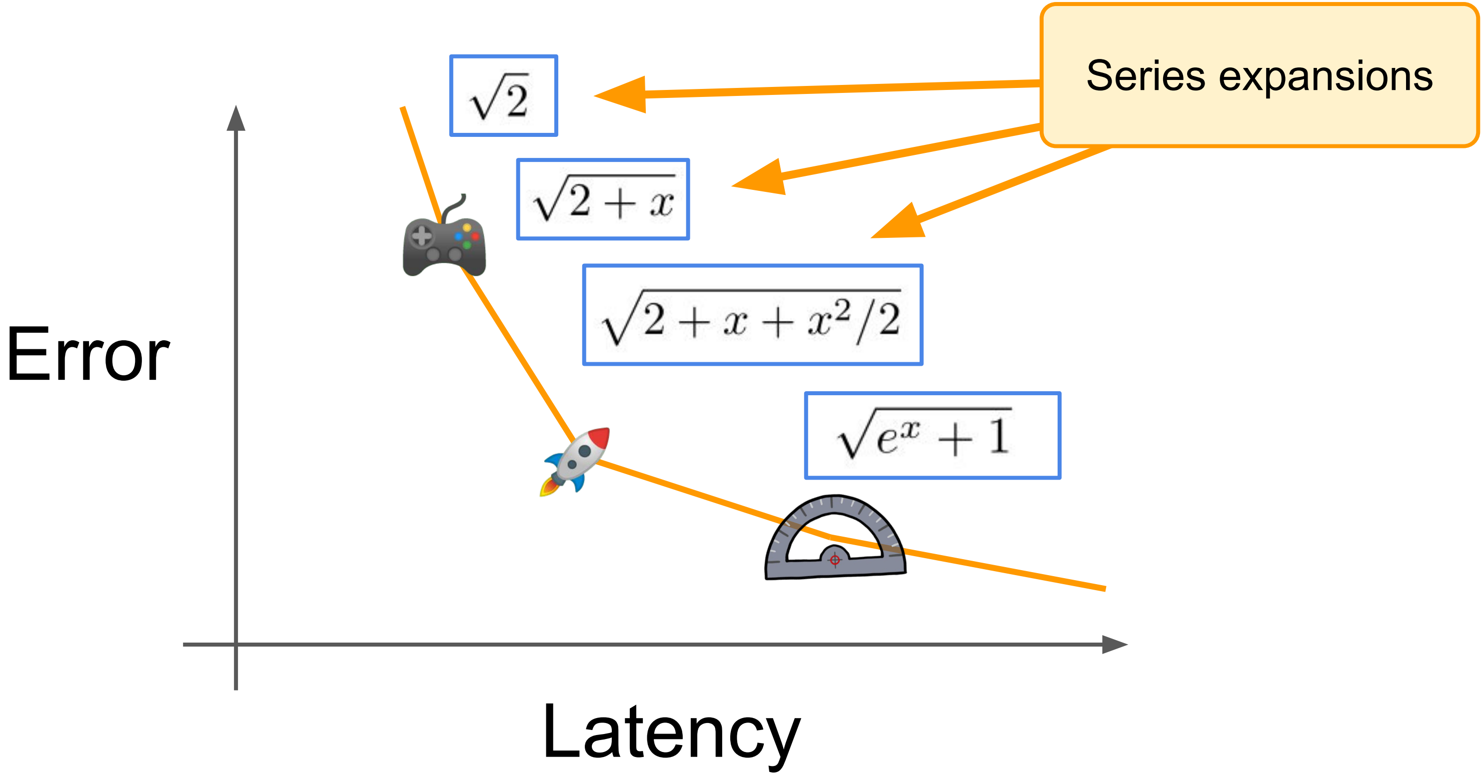

Optimizing in general: precision tuning **OR** rewriting ?

Optimizing in general: precision tuning **OR** rewriting ?

When and how to use?

- Tune then rewrite?
- Rewrite then tune?
- Alternate? Run to fixpoint?
- Share accuracy analyses?

Optimizing in general: precision tuning **AND** rewriting **!**

How to optimize $\sqrt{\frac{e^{2x}-1}{e^x-1}}$ via precision tuning **AND** rewriting !

$$
\begin{aligned}
&\textbf{if} \;\; |x| \leq 0.05 : \\
&\quad \sqrt{2+x} \\
&\textbf{else} : \\
&\quad \sqrt{\langle (e^x+1)_{\text{binary32}} \rangle_{\text{binary64}}}
\end{aligned}
$$

How to optimize $\sqrt{\frac{e^{2x}-1}{e^{x}-1}}$ via precision tuning **AND** rewriting !

Different techniques for different inputs

Sometimes just rewrite

$$
\begin{aligned}
&\textbf{if } |x| \leq 0.05 : \\
&\quad \sqrt{2+x} \\
&\textbf{else} : \\
&\quad \sqrt{\langle (e^{x}+1)_{\text{binary32}} \rangle_{\text{binary64}}}
\end{aligned}
$$

Sometimes rewrite + tune

Optimizing in general: precision tuning **AND** rewriting **!**

Our Result

Combine precision tuning and rewriting to produce a rich set of Pareto-optimal accuracy versus speed trade-offs.

Optimizing in general: precision tuning **AND** rewriting !

Our Result

Combine precision tuning and rewriting to produce a rich set of Pareto-optimal accuracy versus speed trade-offs.

Key Insights:

- Finer-grained interleavings ⇒ better Pareto frontiers
- Precision tuning can be rephrased as a rewriting problem
- “Local Error Analysis” helps both precision tuning and rewriting

Optimizing in general: precision tuning **AND** rewriting !

Ke

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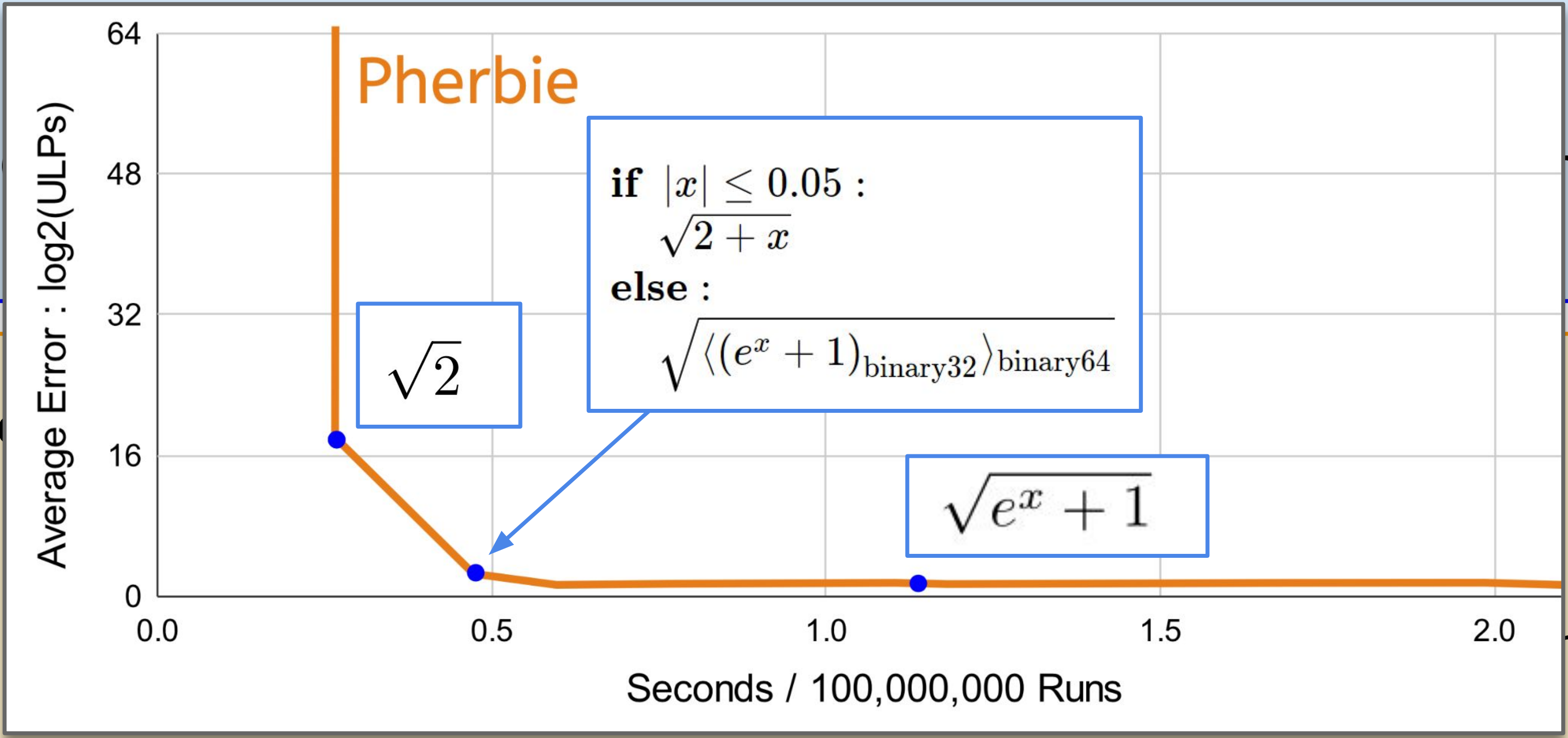
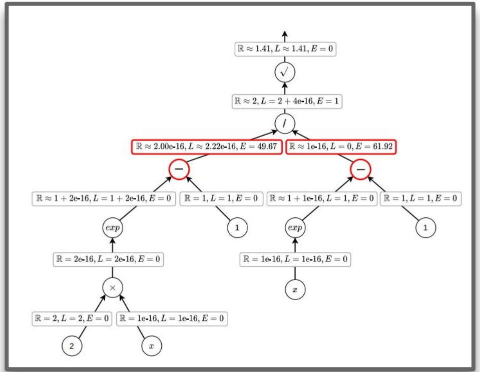
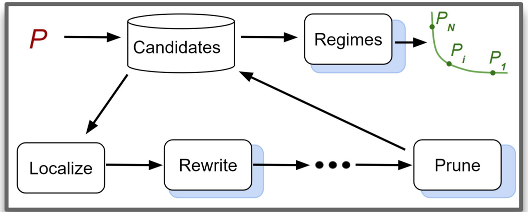


Outline

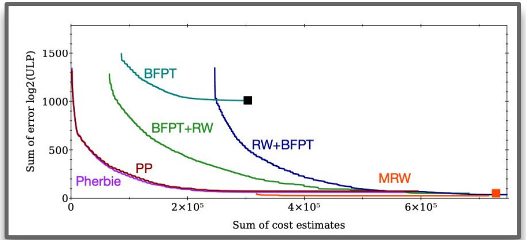
- Herbie: Improving Accuracy via Rewriting
  - Key Insight: local error guides rewriting



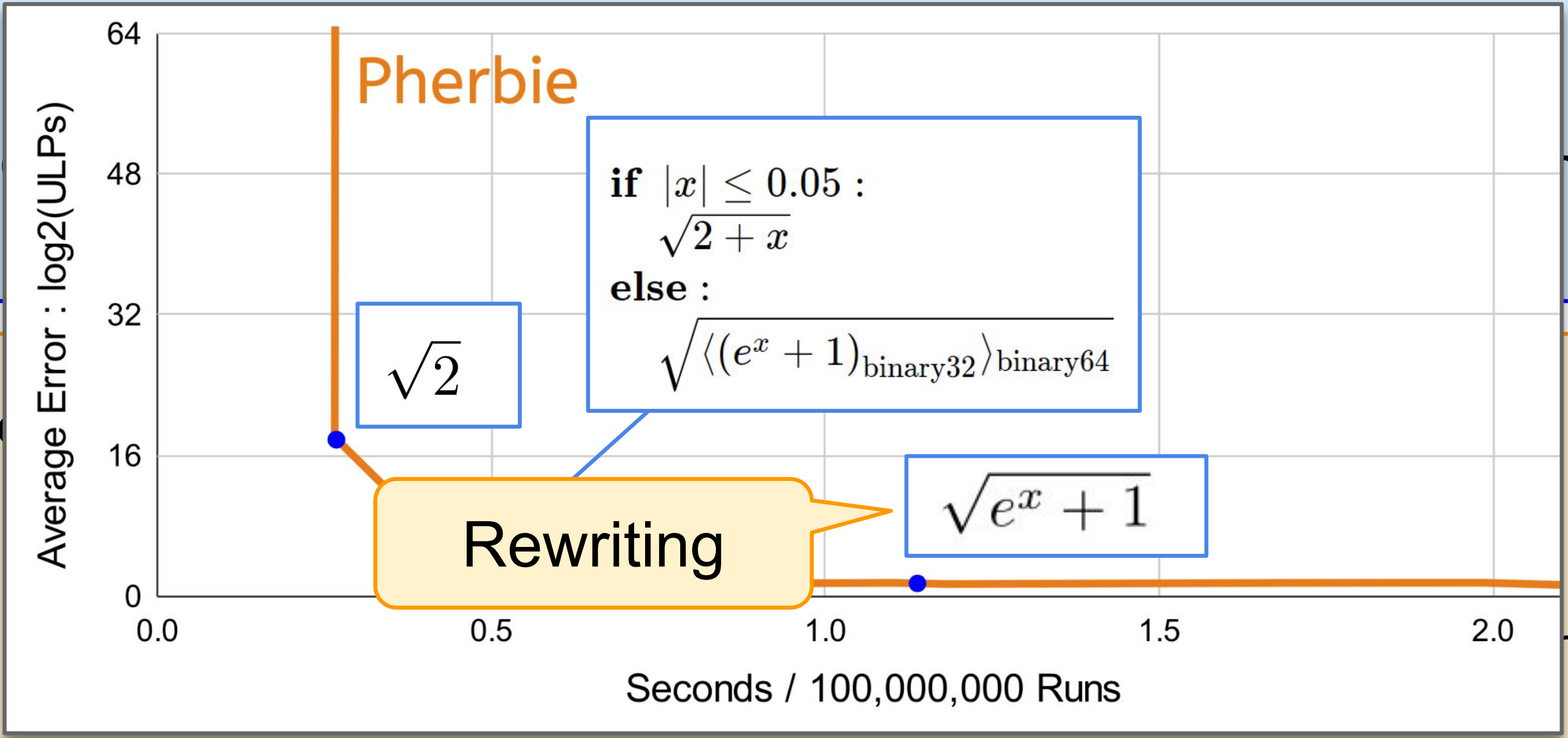
- Pherbie: Extending Herbie with Precision Tuning
  - Key Insight: local error also guides precision tuning!



- Evaluation: Applying Pherbie to Classics + Graphics
  - Key Insight: Finer-grained interleaving → better optimization!



Optimizing in general: precision tuning **AND** rewriting !



Developed continuously since 2015

Improves Accuracy Automatically

Rewriting Only

Input

$$\sqrt{\frac{e^{2x} - 1}{e^{x} - 1}}$$

Rewriting →

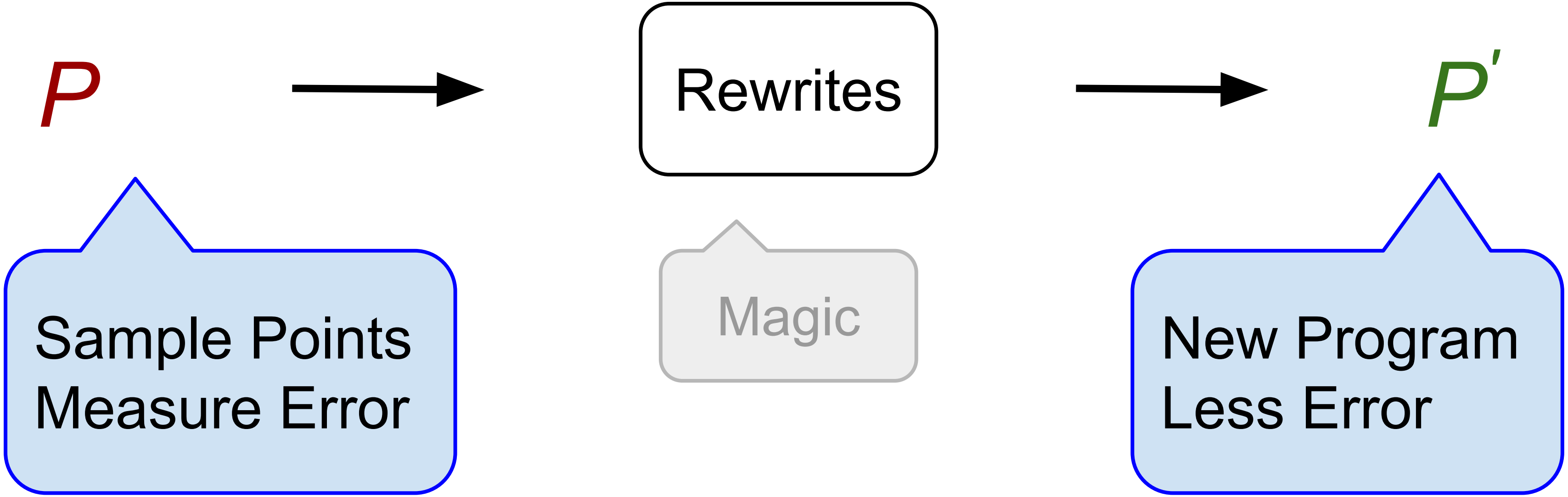
Output

$$\sqrt{e^{x} + 1}$$

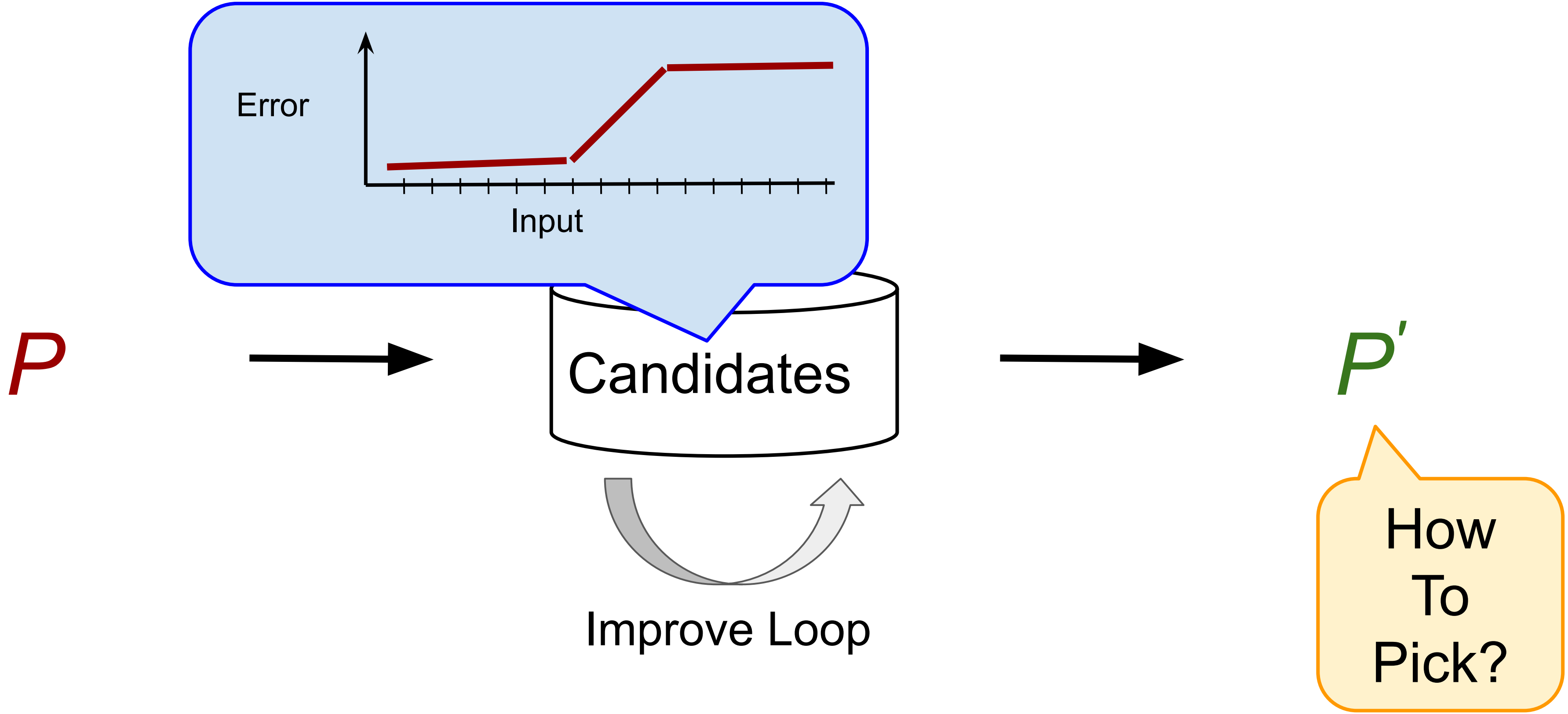
Developed continuously since 2015

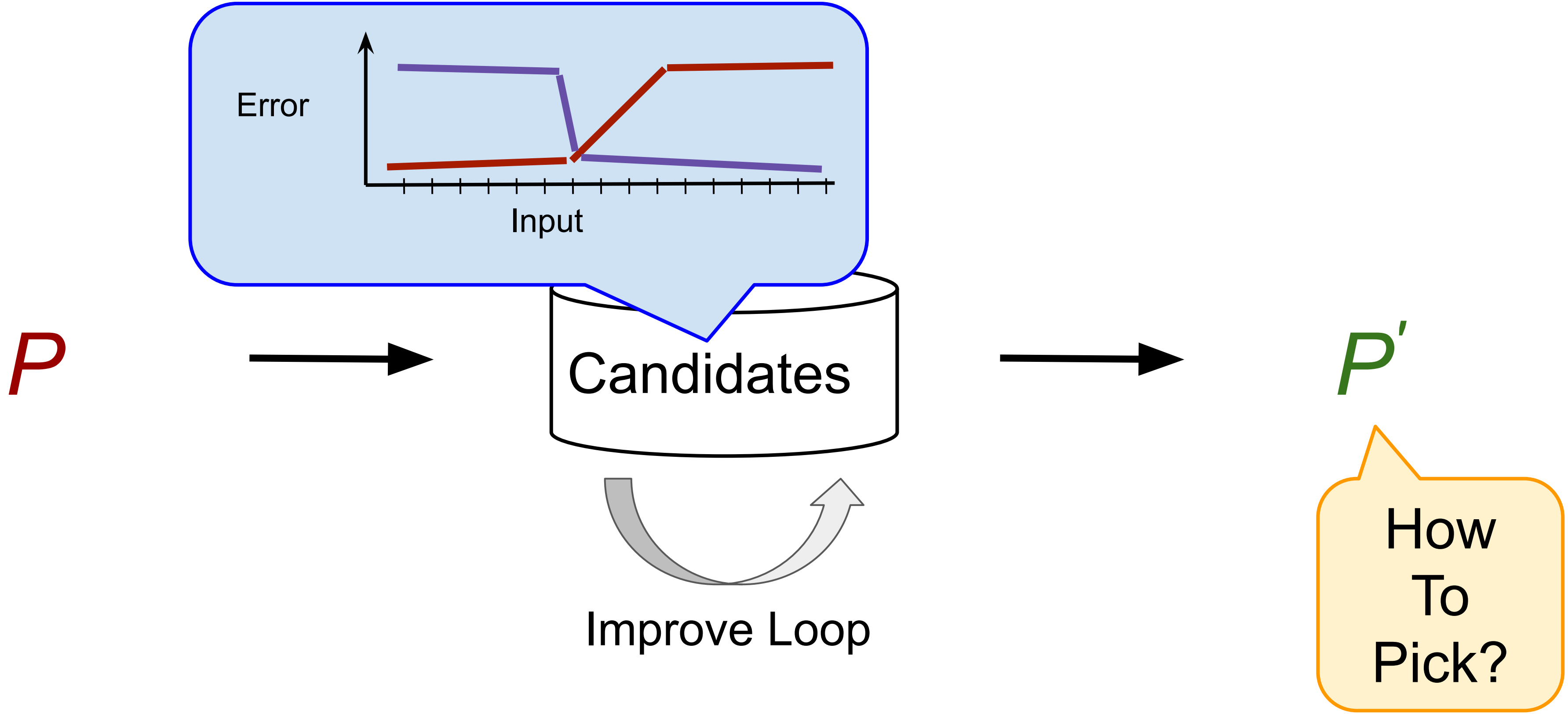
Improves Accuracy Automatically

Rewriting Only



P
Rewrites
P'
Sample Points
Measure Error
Magic
New Program
Less Error


Error
Input
P
Candidates
P'
Improve Loop
How To Pick?



Error
Input
P
Candidates
P′
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How
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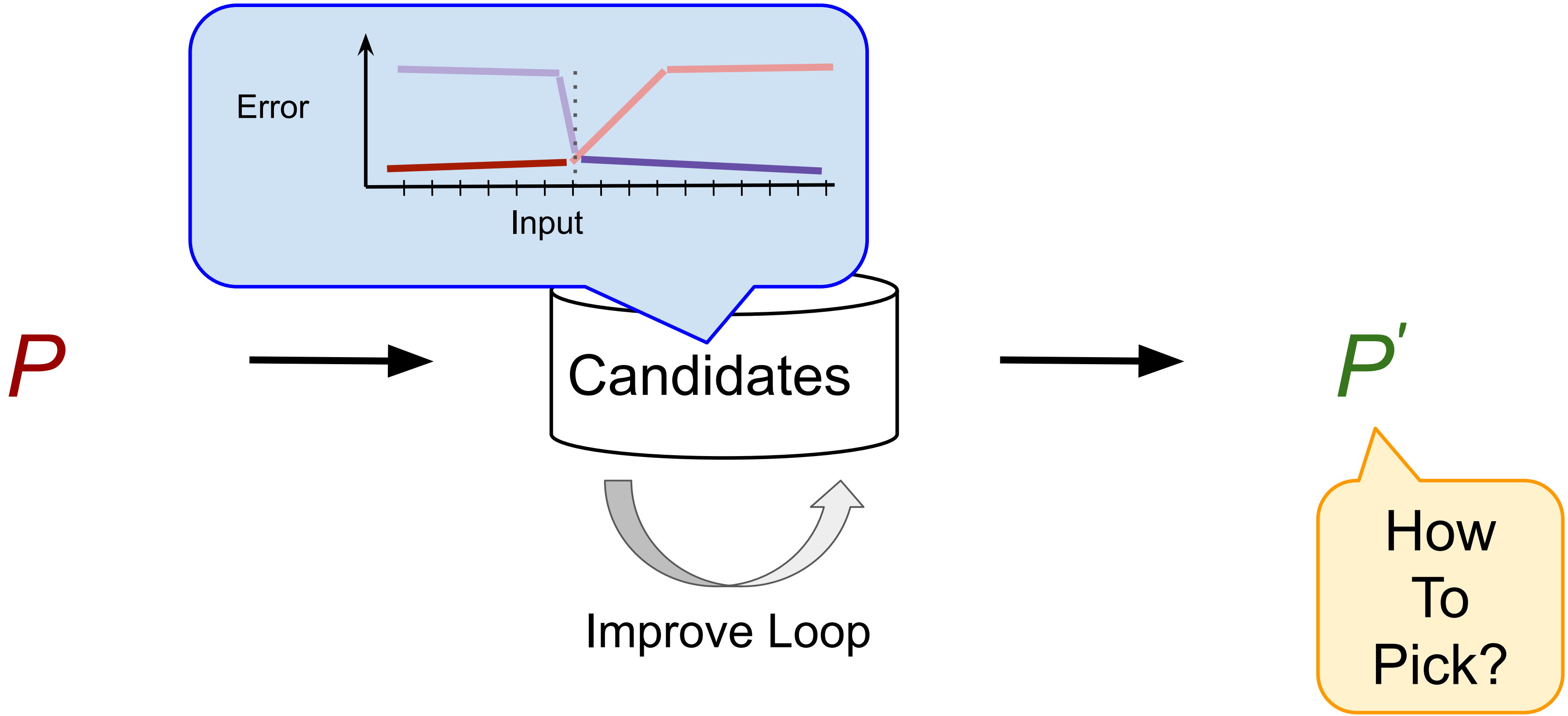
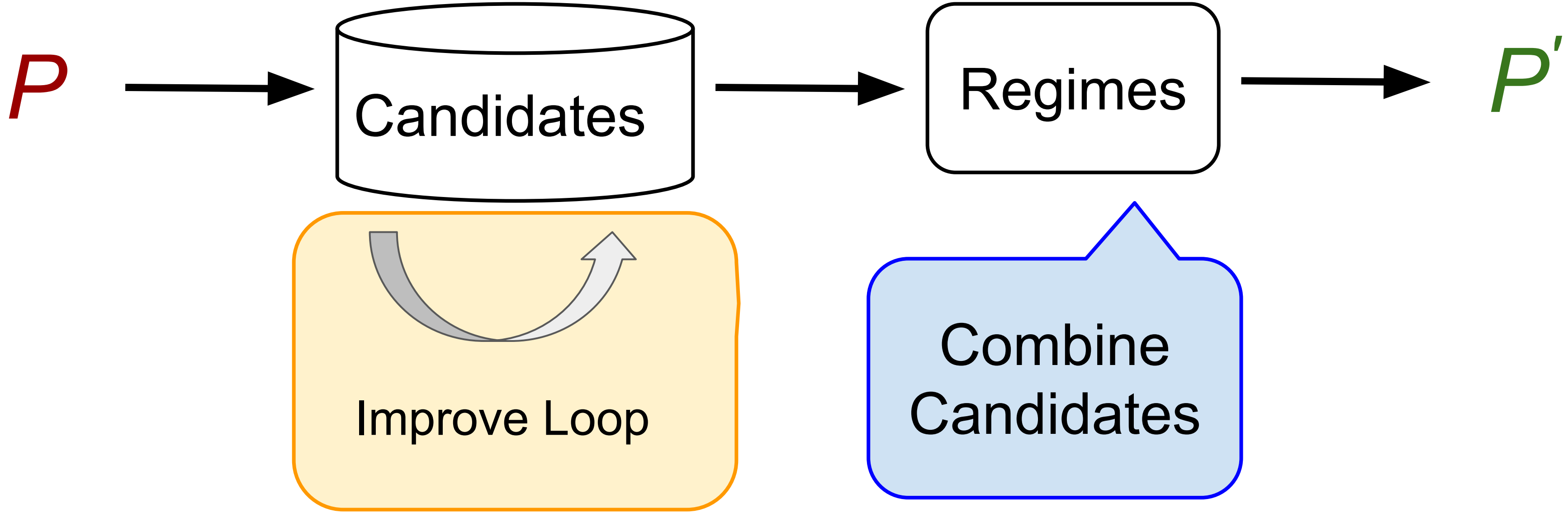
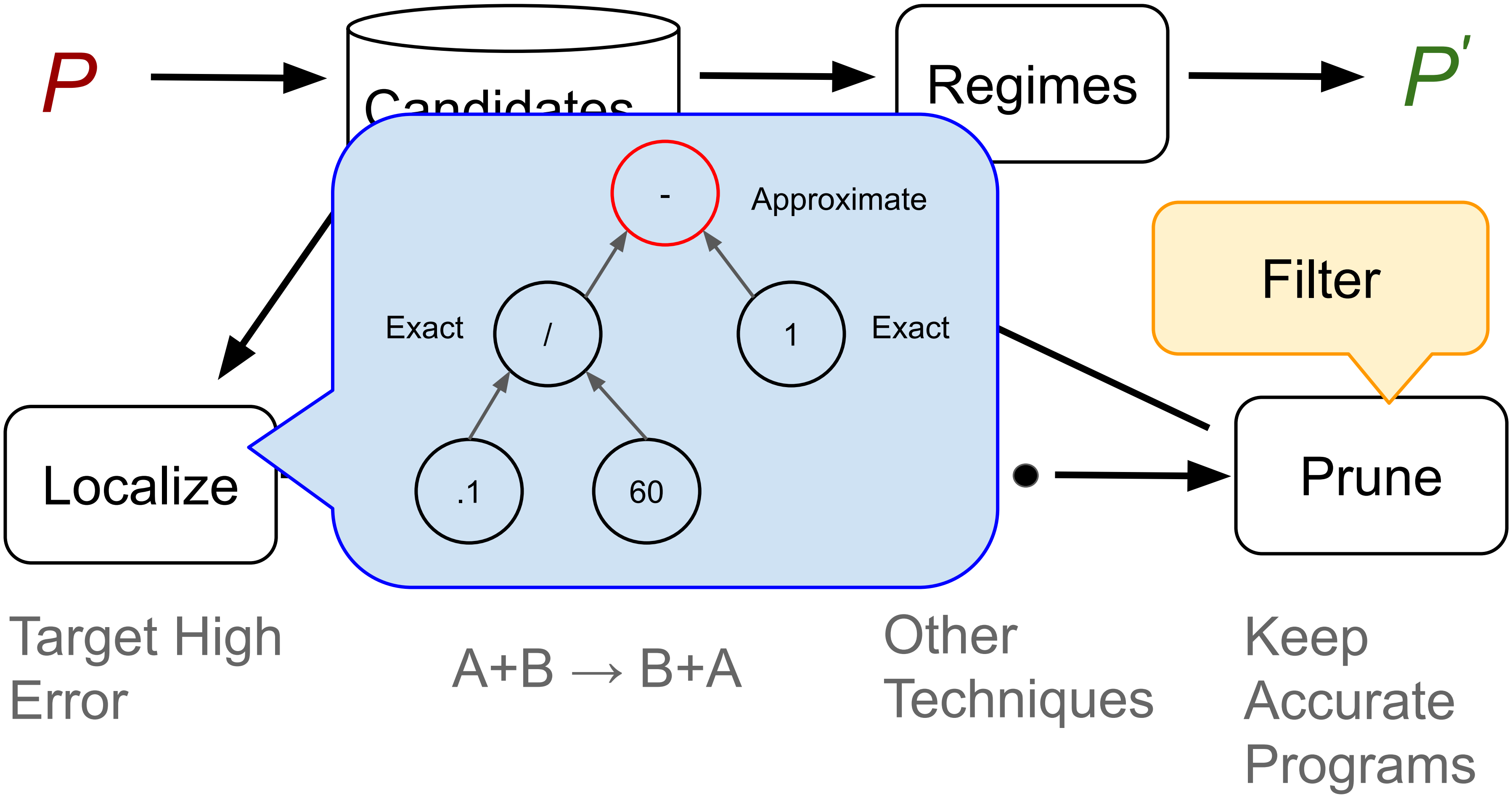
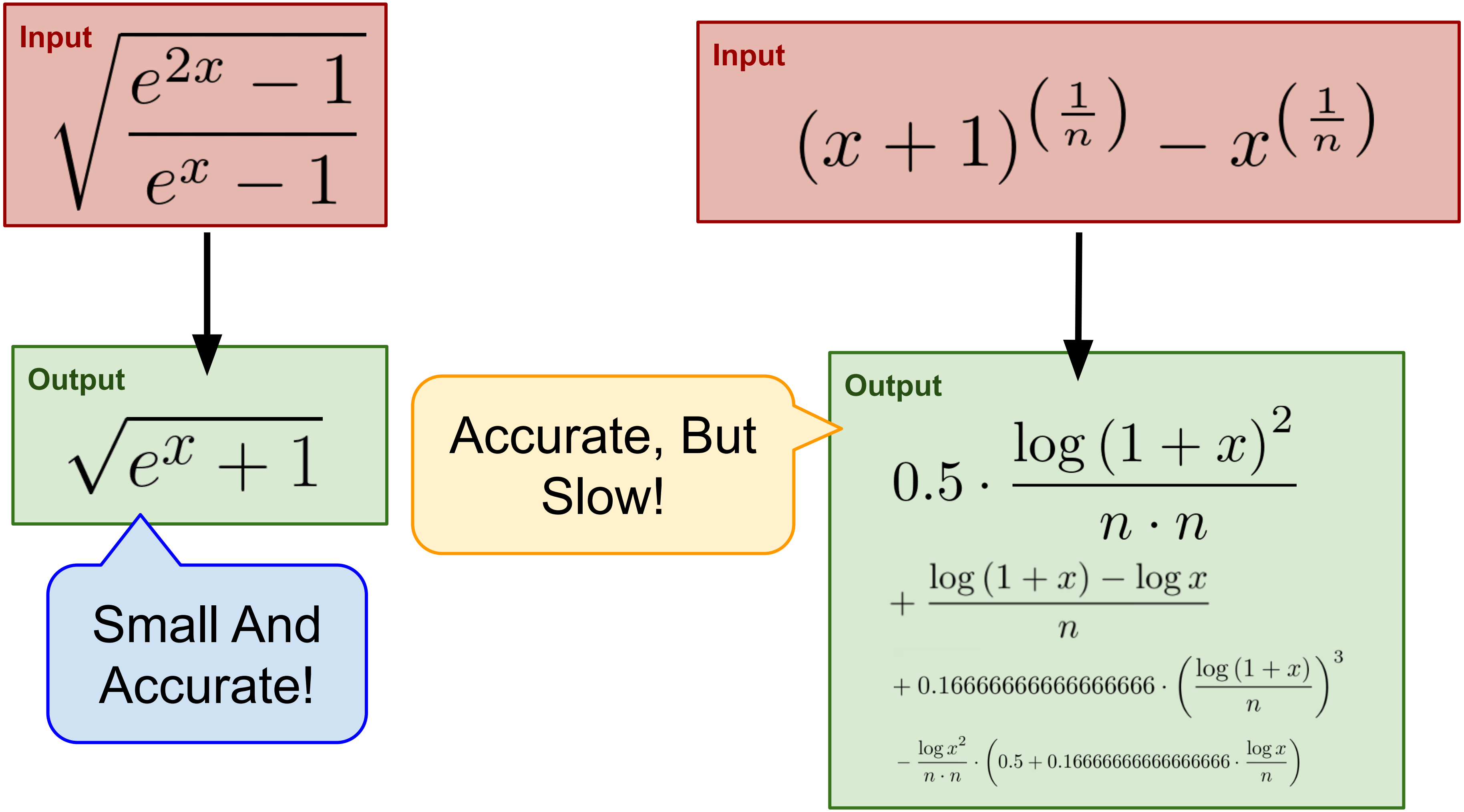


Error
Input
P
Candidates
P′
Improve Loop
How To Pick?


P
Candidates
Regimes
P'
Improve Loop
Combine
Candidates



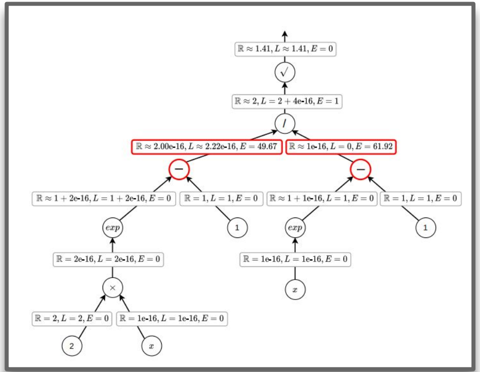
P
Candidates
Regimes
P′
Approximate
-
Exact
/
1
Exact
.1
60
Filter
Localize
Prune
Target High Error
A+B → B+A
Other Techniques
Keep Accurate Programs


Input
$\sqrt{\frac{e^{2x} - 1}{e^x - 1}}$
Output
$\sqrt{e^x + 1}$
Small And Accurate!
Input
$(x+1)^{\left(\frac{1}{n}\right)} - x^{\left(\frac{1}{n}\right)}$
Accurate, But Slow!
Output
$0.5 \cdot \frac{\log{(1+x)}^2}{n \cdot n} + \frac{\log{(1+x)} - \log x}{n} + 0.16666666666666666 \cdot \left(\frac{\log{(1+x)}}{n}\right)^3 - \frac{\log x^2}{n \cdot n} \cdot \left(0.5 + 0.16666666666666666 \cdot \frac{\log x}{n}\right)$

Outline

✓ Herbie: Improving Accuracy via Rewriting

- Key Insight: local error guides rewriting



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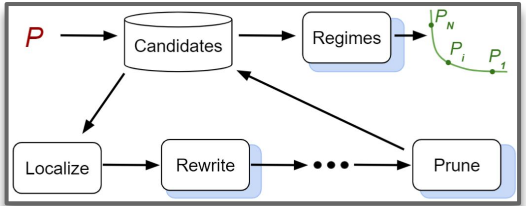

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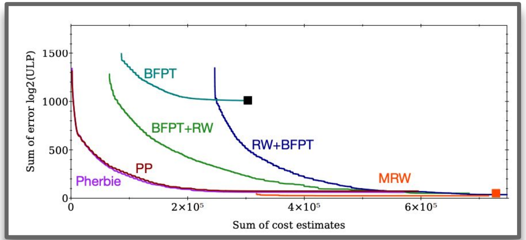
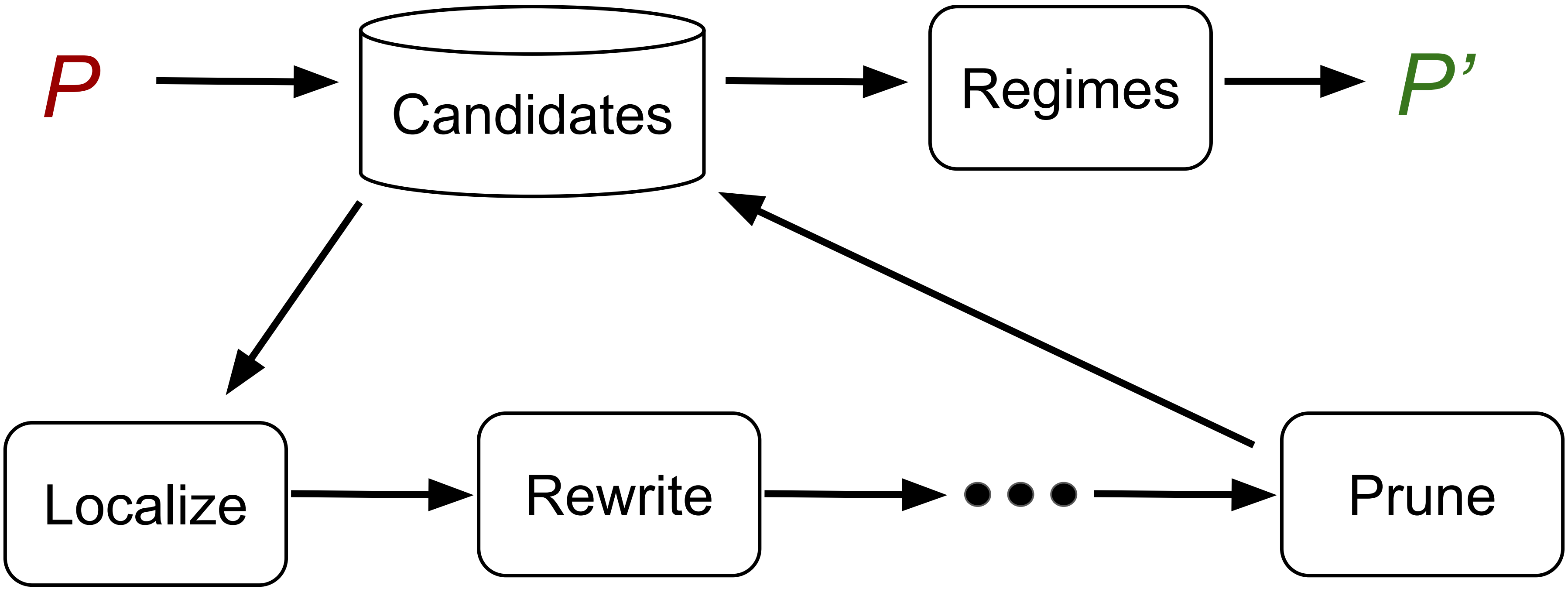
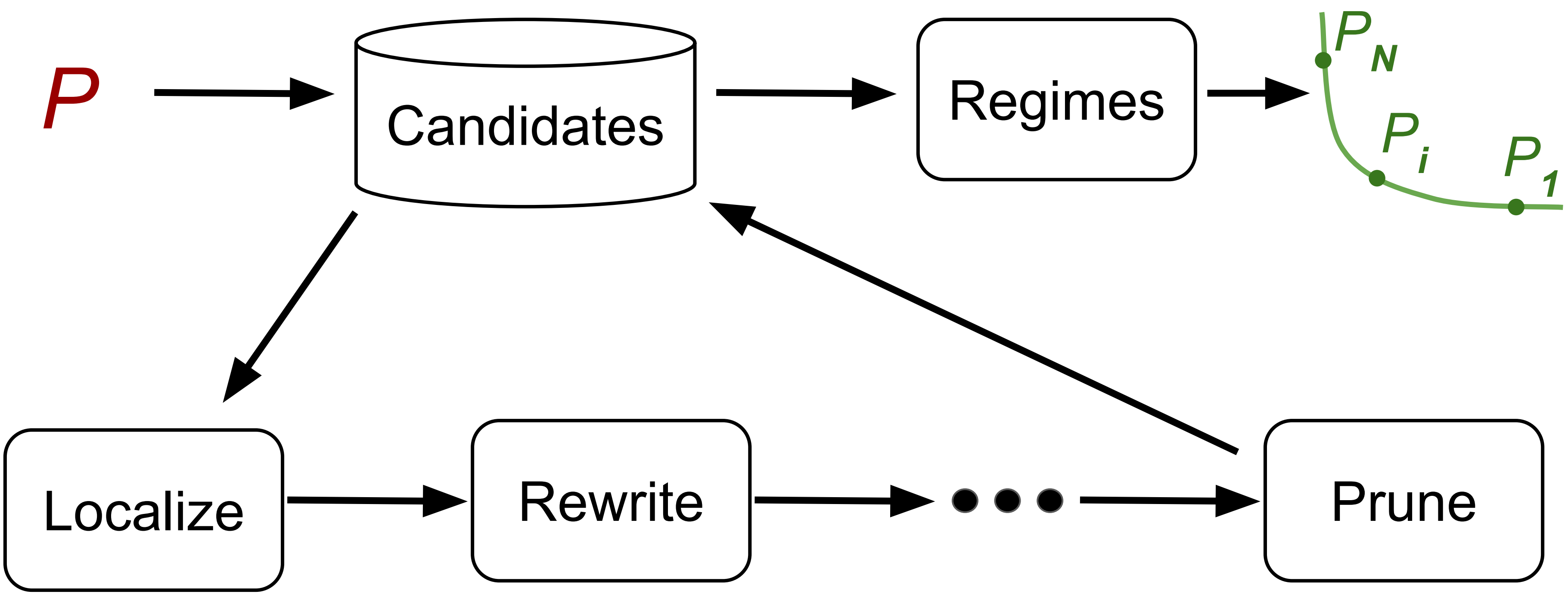


Pherbie Starting Point: Herbie



Pherbie: Extending Herbie to Combine Tuning + Rewriting



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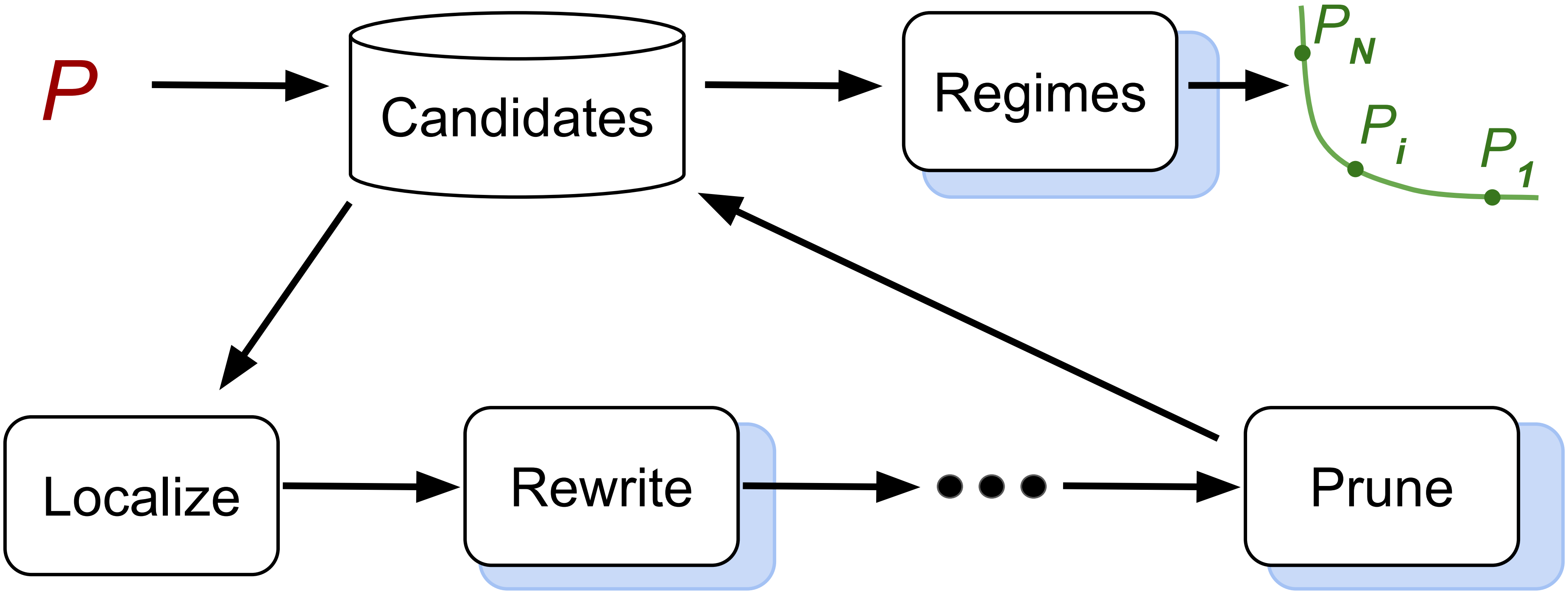

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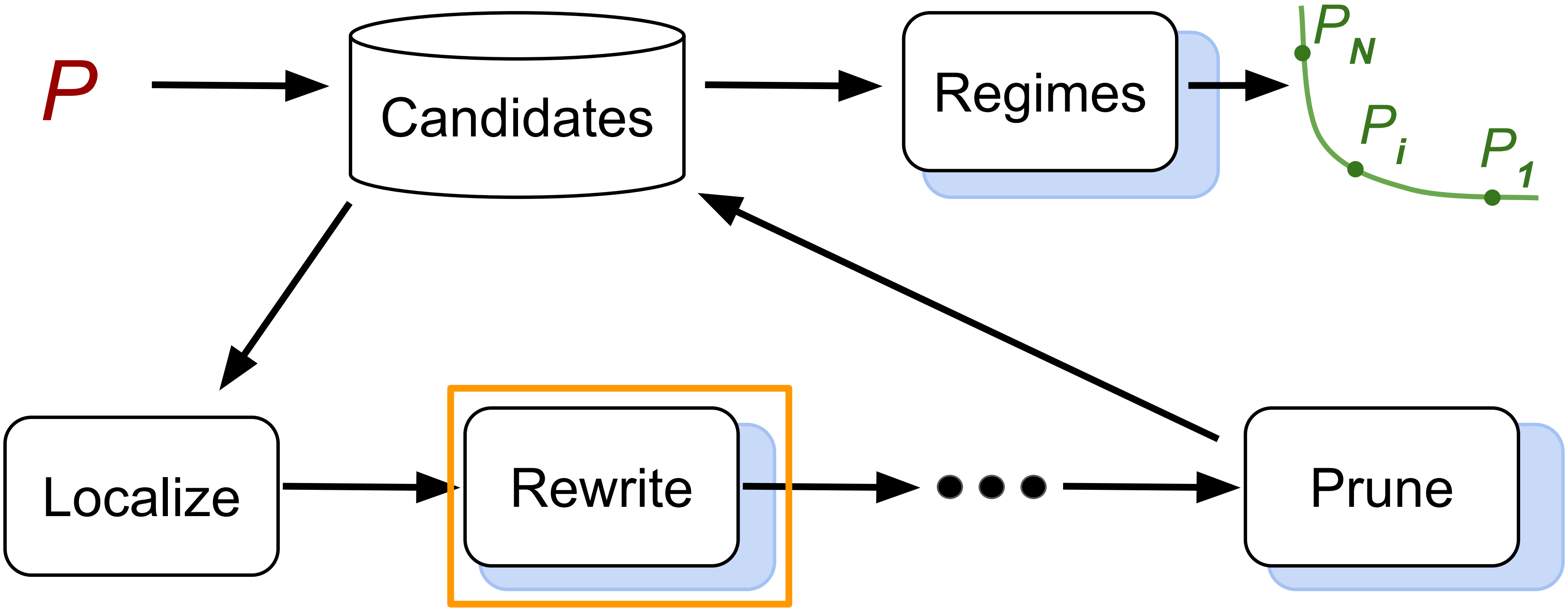
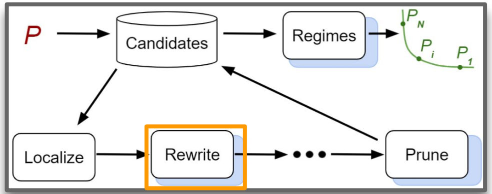


Pherbie: Precision Rewrites

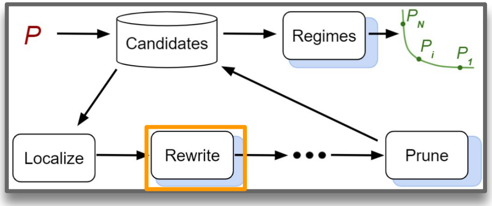


Single global precision

Herbie $$(a + b) \;/\; c \;\Rightarrow\; (a \;/\; c) + (b \;/\; c)$$

Pherbie

Pherbie: Precision Rewrites

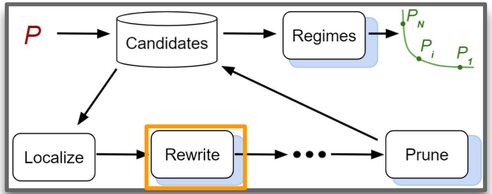


Herbie $(a + b)\ /\ c \Rightarrow (a\ /\ c) + (b\ /\ c)$

Single global precision

Pherbie $(a +_{\mathrm{f64}} b)\ /_{\mathrm{f64}}\ c \Rightarrow (a\ /_{\mathrm{f64}}\ c) +_{\mathrm{f64}} (b\ /_{\mathrm{f64}}\ c)$

Precision-specific operators



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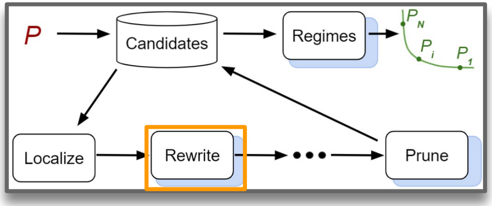
Precision-specific operators

$$(x)_p \ \Rightarrow \ \mathrm{cast}_p \, (x)_q$$

Precision rewrites



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Herbie $$(a + b) \;/\; c \;\Rightarrow\; (a \;/\; c) + (b \;/\; c)$$

Rewriting

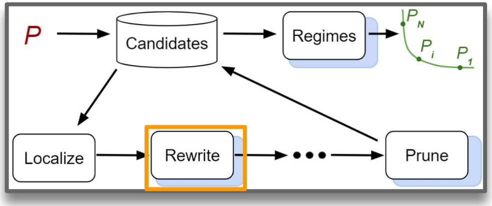
Pherbie $$(a +_{\mathrm{f64}} b) \;/_{\mathrm{f64}}\; c \;\Rightarrow\; (a \;/_{\mathrm{f64}}\; c) +_{\mathrm{f64}} (b \;/_{\mathrm{f64}}\; c)$$

Rewriting

$$(x)_p \;\Rightarrow\; \mathrm{cast}_p\,(x)_q$$

Precision tuning

Pherbie: Precision Rewrites



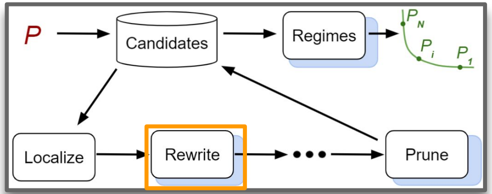
Herbie $$(a + b) \;/\; c \;\Rightarrow\; (a \;/\; c) + (b \;/\; c)$$

Pherbie

Pherbie can use the same rewriting machinery as Herbie!

$$(x)_p \;\Rightarrow\; \mathrm{cast}_p\,(x)_q$$

Pherbie: Precision Rewrites



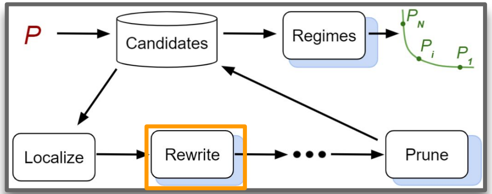
Herbie $$(a + b) \,/\, c \;\Rightarrow\; (a \,/\, c) + (b \,/\, c)$$

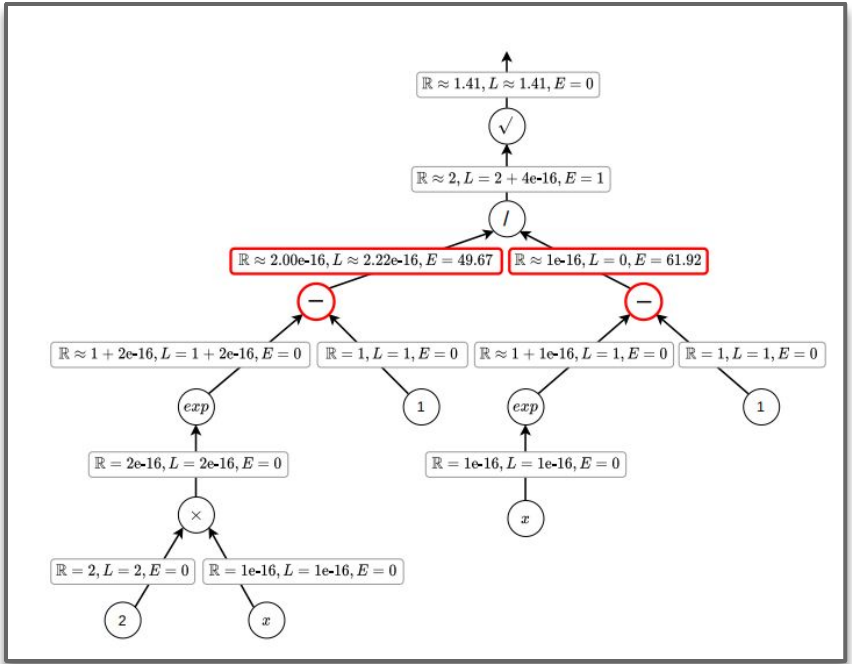
Pherbie

Pherbie can use the same rewriting machinery as Herbie!

But where should Pherbie apply precision rewrites?

Pherbie: Guide Tuning w/ Local Error





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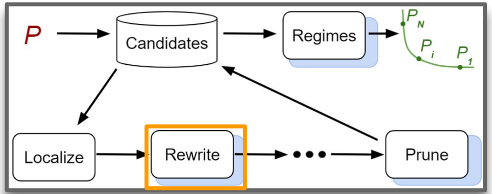
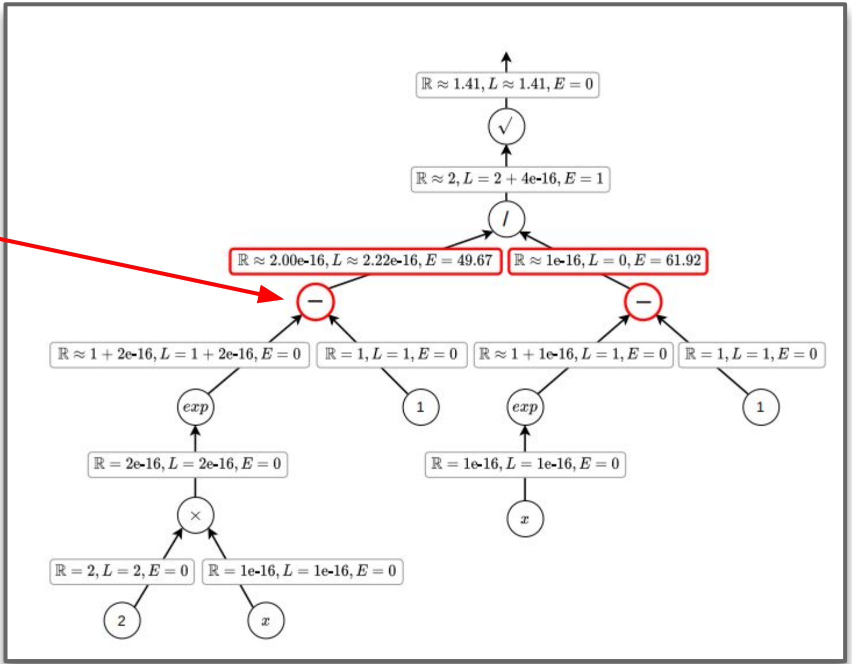


- Rewriting to increase precision at locations w/ *high local error* improves accuracy.



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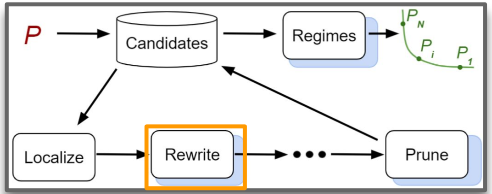
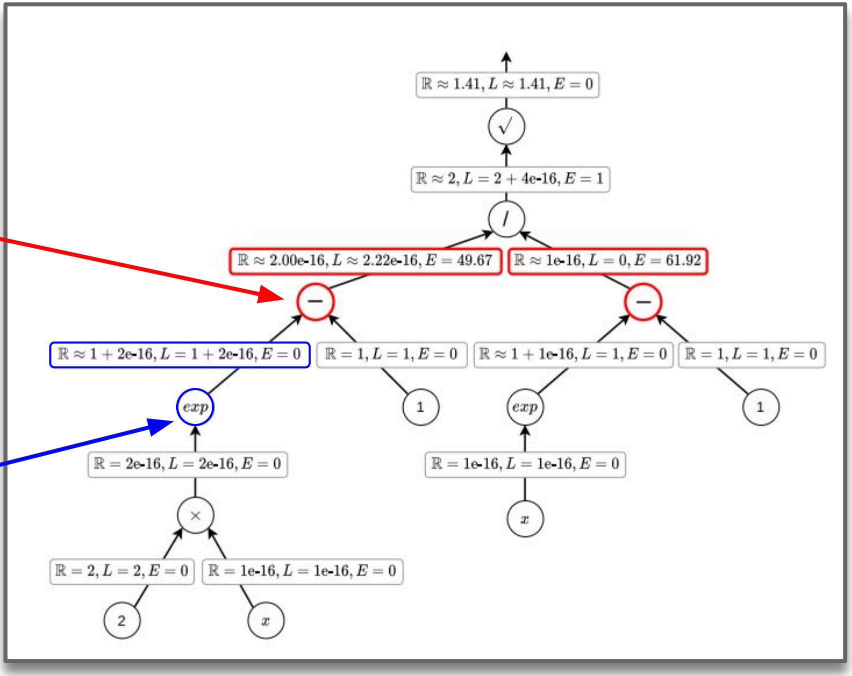
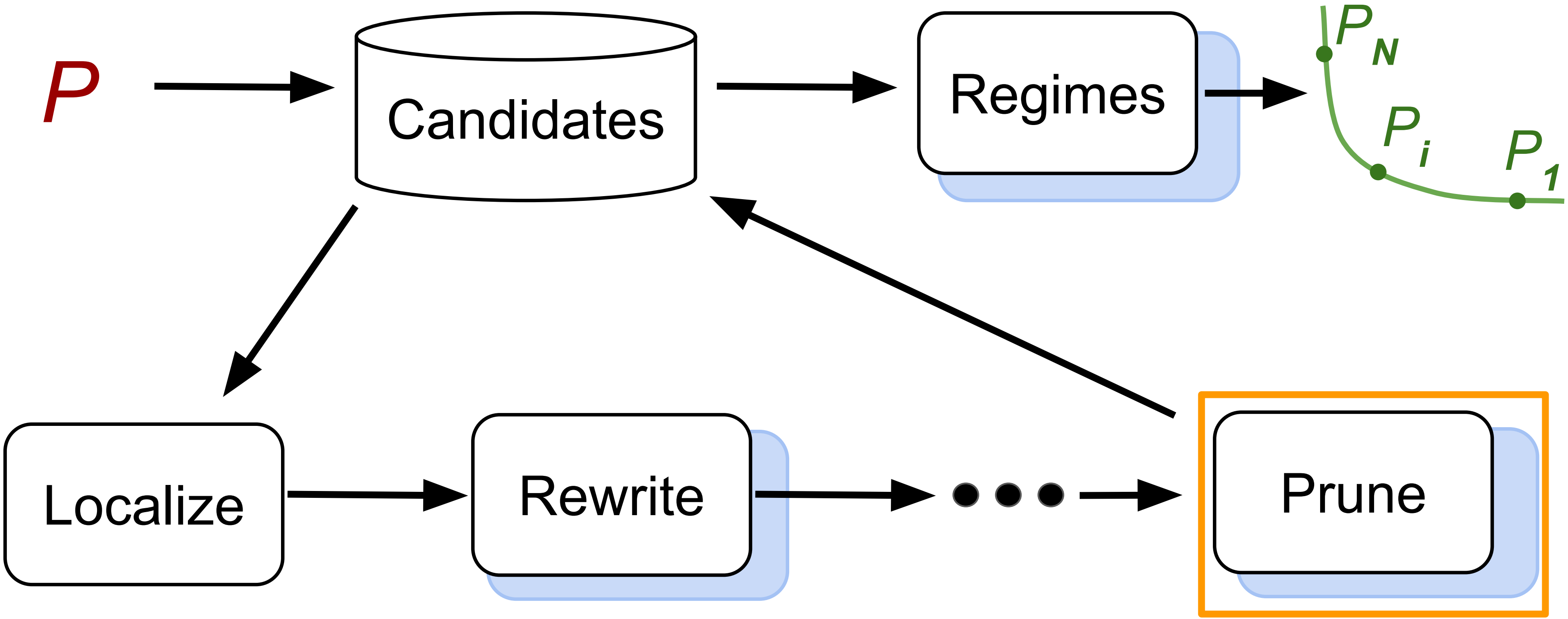


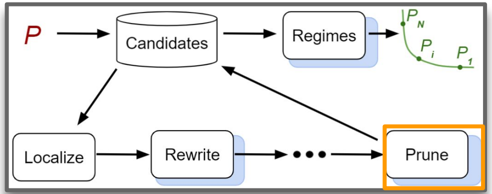
- Rewriting to increase precision at locations w/ *high local error* improves accuracy.

- Rewriting to decrease precision at locations w/ *low local error* improves speed.



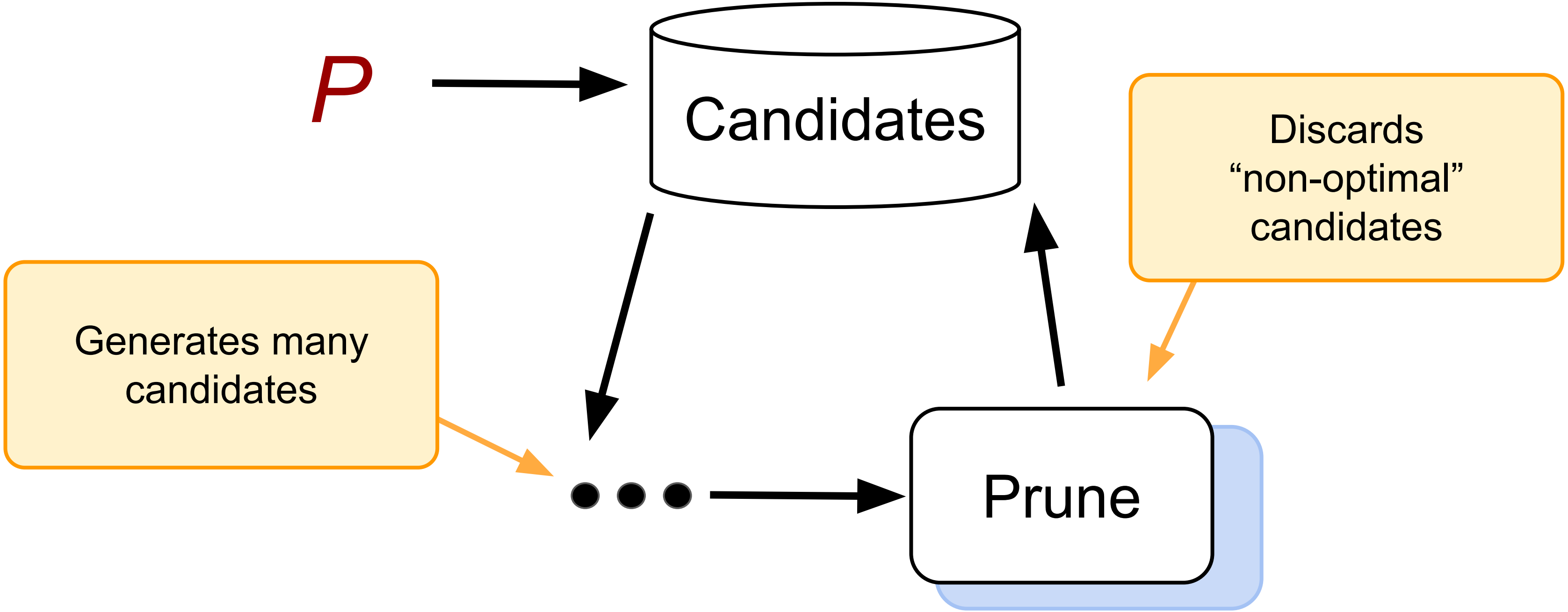
Pherbie: Extending Herbie to Combine Tuning + Rewriting



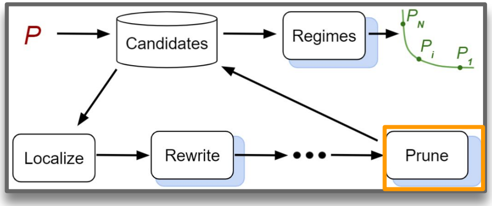


Pherbie: Pruning

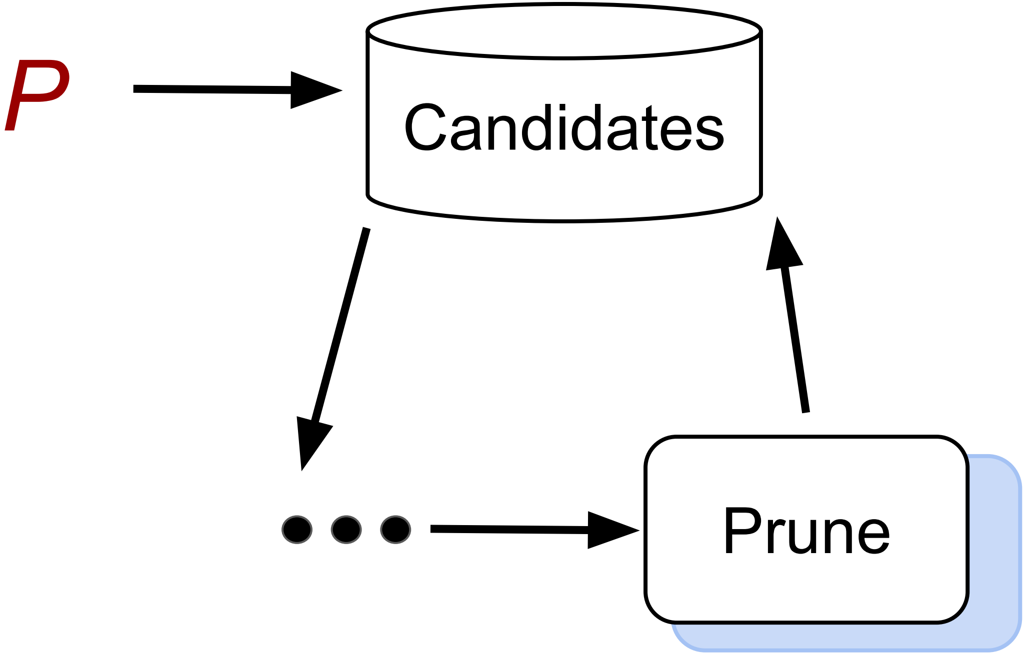
Pruning in general



Pherbie: Pruning



Pruning in Herbie:



Criteria

Must be more accurate than every other expression on at least one sampled point

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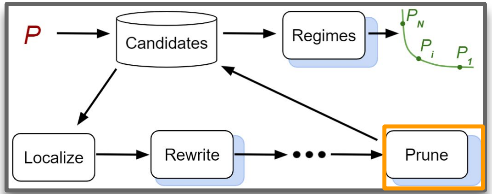

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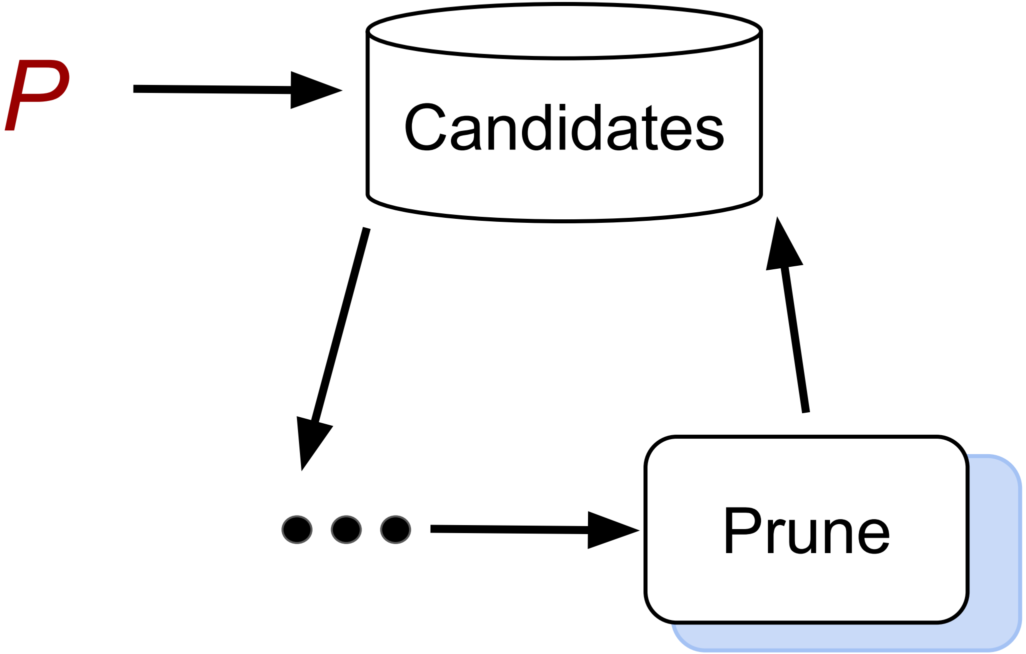


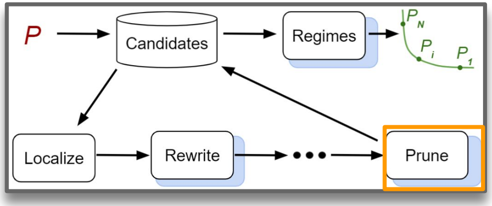
Criteria

Must be more accurate than every other expression on at least one sampled point

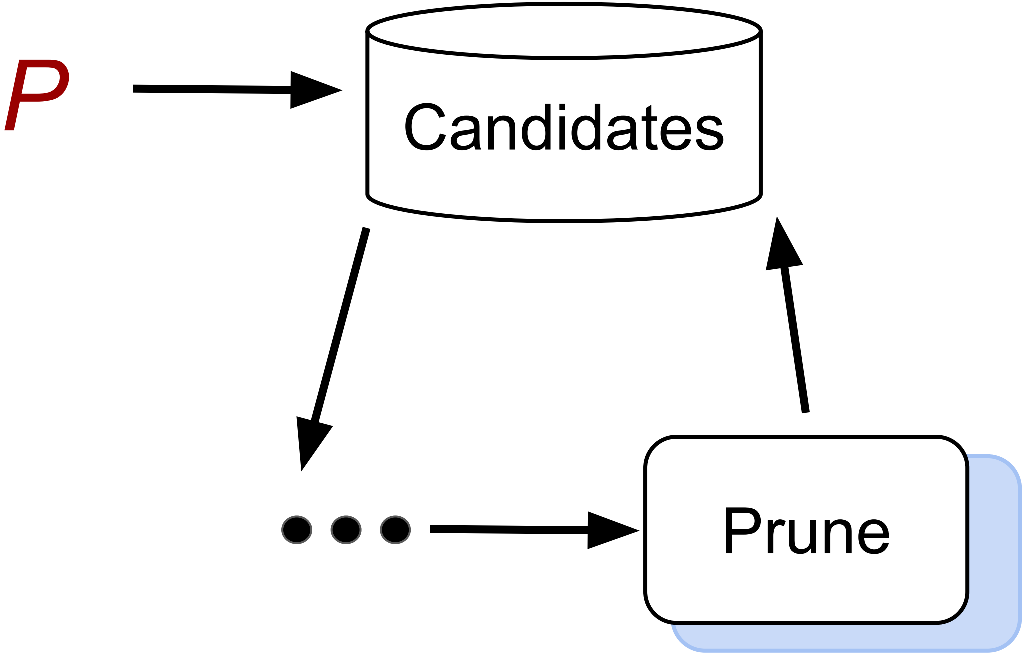
Use in Pherbie?

Accuracy only ⇒ slow expressions

Pherbie: Pruning



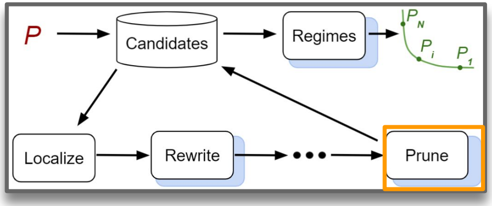
Pruning in Pherbie:



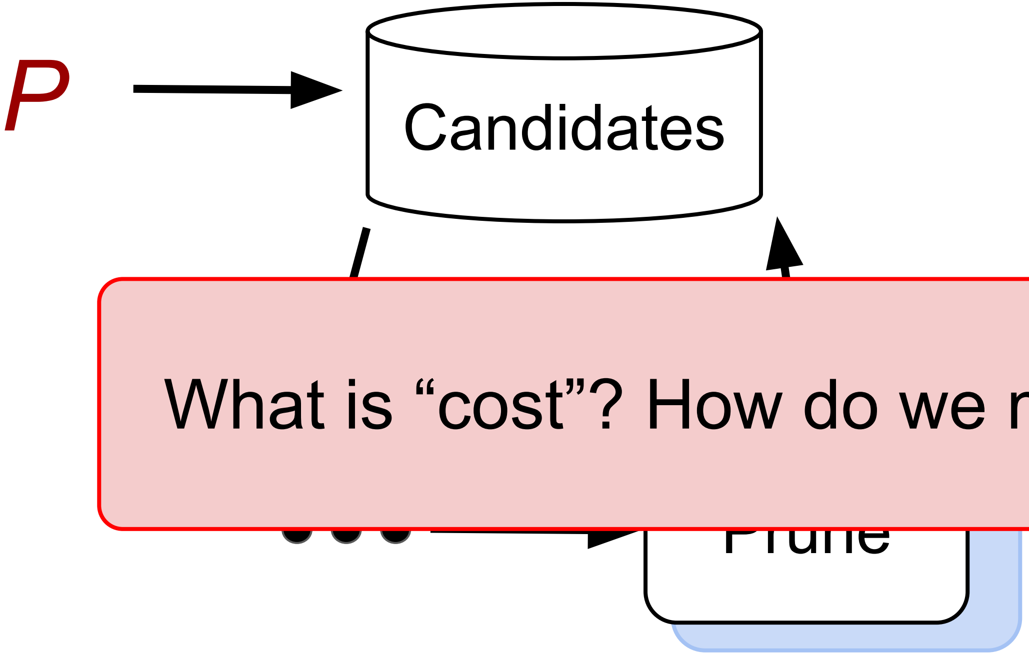
Criteria

Must be more accurate on at least one sampled point than every other expression *at or below the cost of the candidate*

Pherbie: Pruning



Pruning in Pherbie:

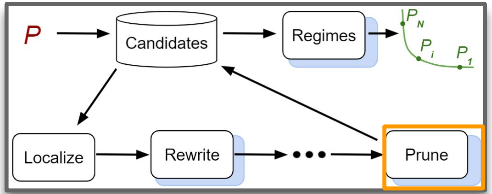


Criteria

Must be more accurate on at least one sampled point than every other expression *at or below the cost of the candidate*

What is “cost”? How do we measure it?

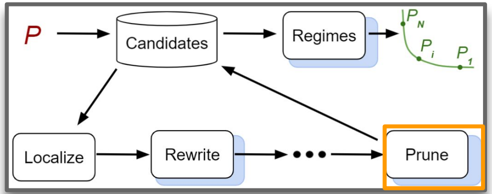
Pherbie: Pruning



What is "cost"? How do we measure it?

Too expensive to measure precise latency of each candidate

- Need to evaluate candidate many times to get accurate estimator
- Pherbie produces thousands of candidates

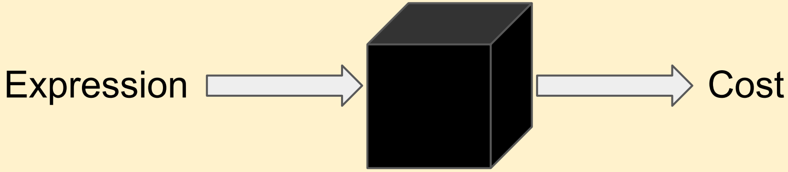


Pherbie: Pruning

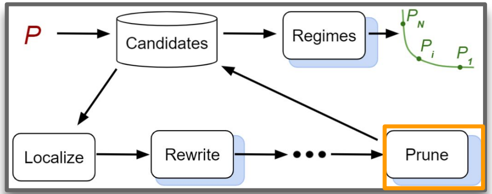
What is “cost”? How do we measure it?

Key Insight: Only need relative speed comparison → use a simple cost model!

- Quickly estimates latency
- Sufficient for relative ordering of candidates



Pherbie: Pruning



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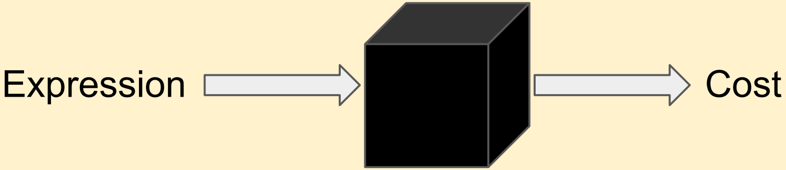
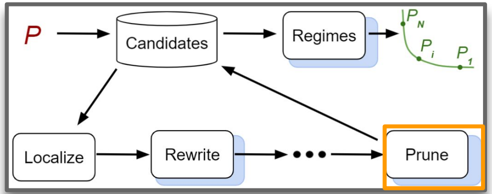


Example cost model:

- Operators assigned a cost:
  - Arithmetic: low number (1)
  - Library functions: large number (100)
- Multiply operator cost by bitwidth of representation
- Conditionals: branch conditions cost + largest branch cost

Pherbie: Pruning



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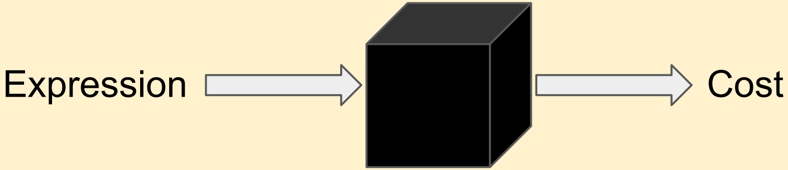
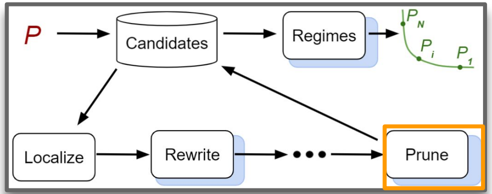


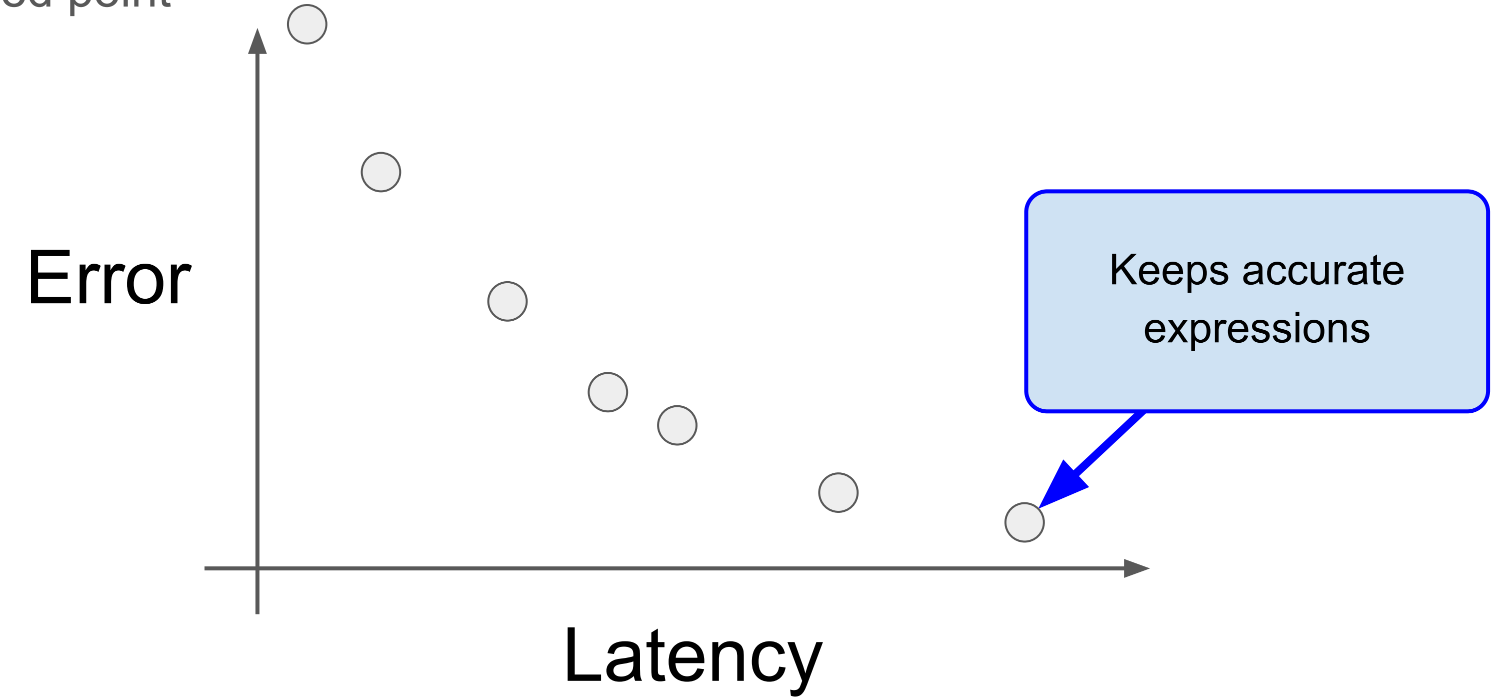
Cost models in general

- Simple cost models are good enough
- Better cost models exist
- Pherbie is modular, so users can plug and play

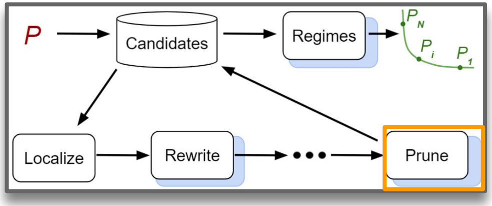
Pherbie: Pruning



At each sampled point



Pherbie: Pruning



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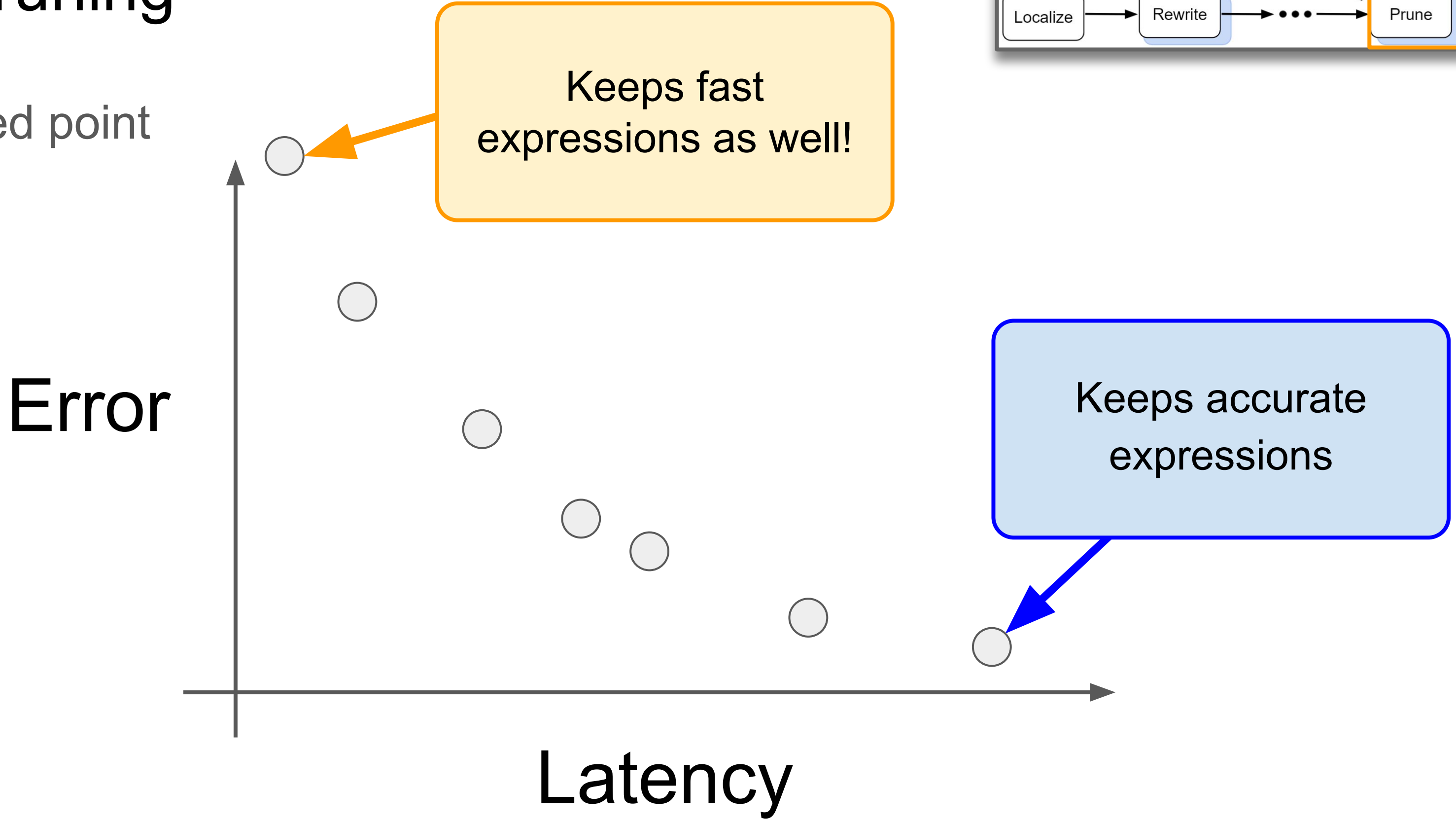

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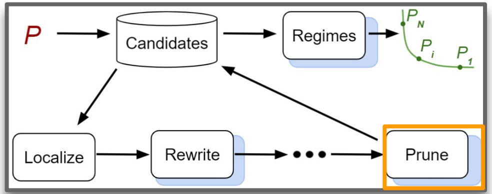

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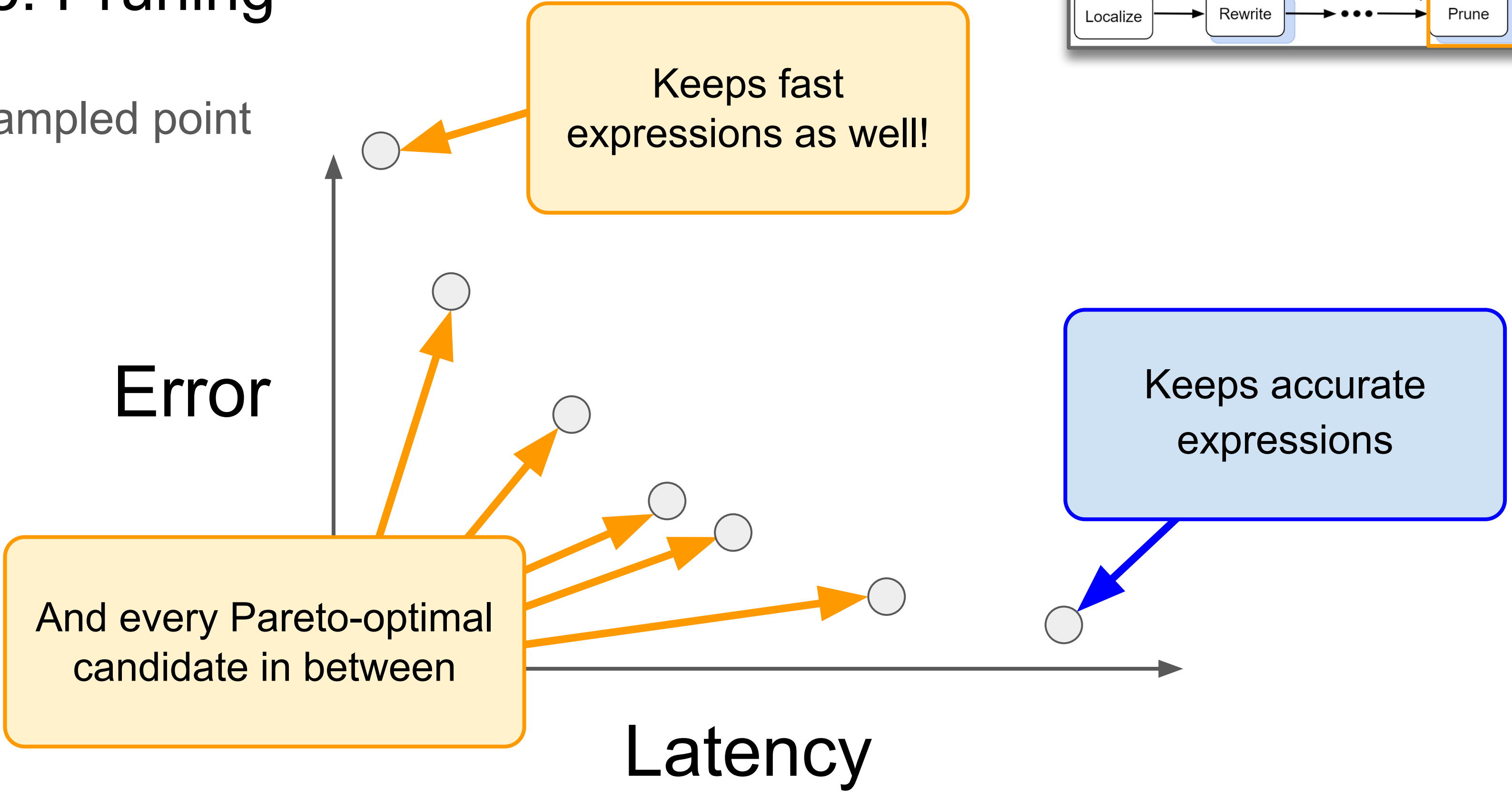
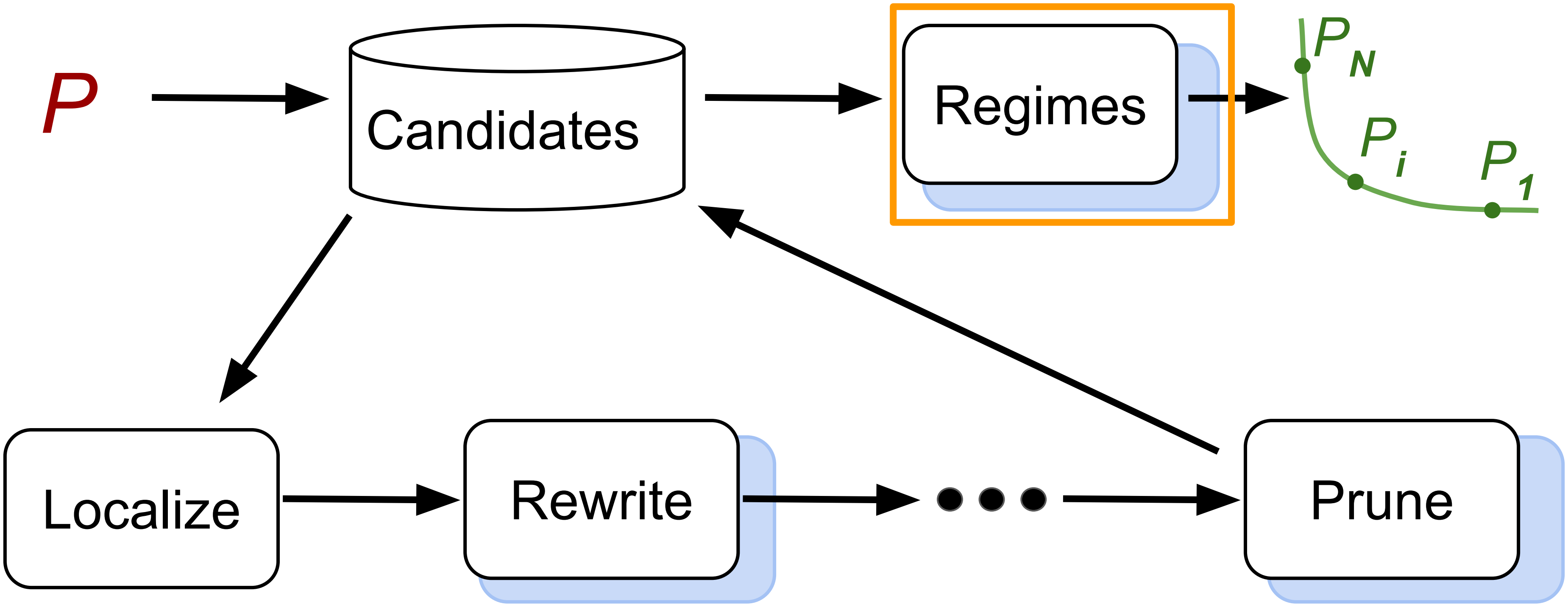
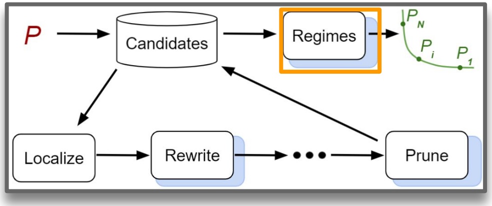


Pherbie: Extending Herbie to Combine Tuning + Rewriting

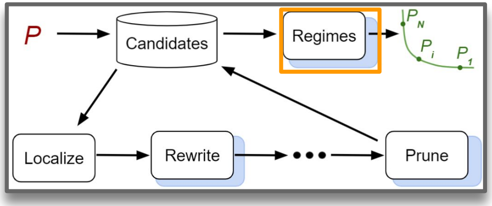




Pherbie: Regimes

Pherbie: accuracy and cost

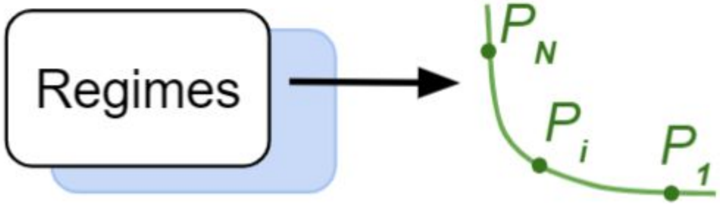
- Need to produce a Pareto frontier!
- Iteratively run Herbie's regimes algorithm on subset of candidates



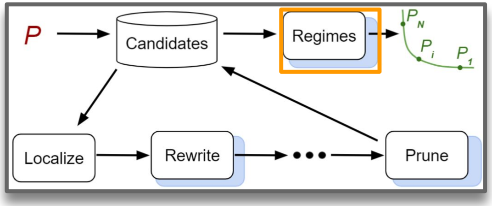
Pherbie: Regimes

Pherbie regimes algorithm

1. Run Herbie regimes algorithm on subset cheaper than cost bound

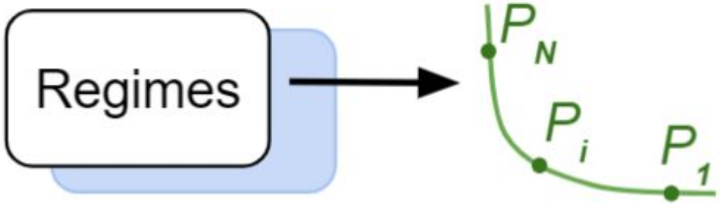


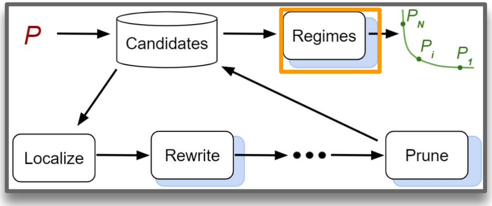
Pherbie: Regimes



Pherbie regimes algorithm

1. Run Herbie regimes algorithm on subset cheaper than cost bound
2. Decrease cost bound so next iteration produces *different* candidate

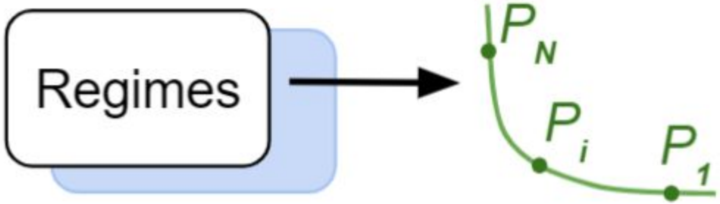




Pherbie: Regimes

Pherbie regimes algorithm

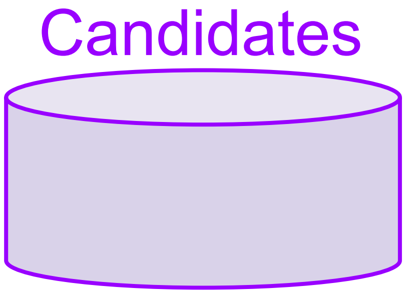
1. Run Herbie regimes algorithm on subset cheaper than cost bound
2. Decrease cost bound so next iteration produces *different* candidate
3. Repeat until no candidate is below cost bound

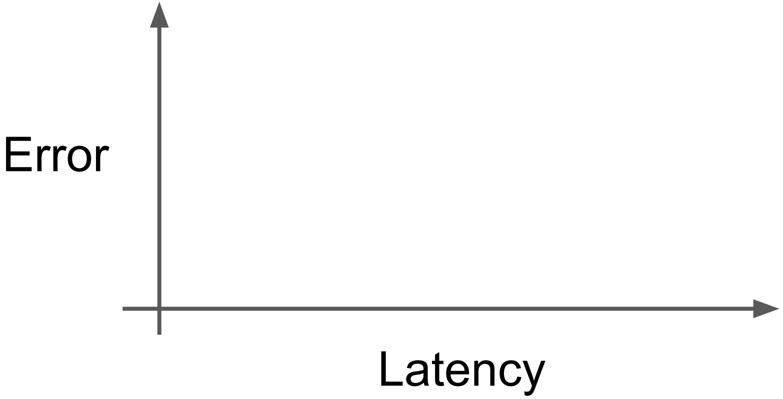


Pherbie Regimes Example : Iter 1 / 5

$$\sqrt{\frac{e^{2x} - 1}{e^{x} - 1}}$$

```
while 0 < |Candidates| :
    p = ExtractMinError(Candidates)
    Candidates.removeAboveCost(p)
```





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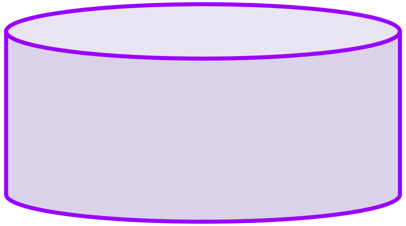
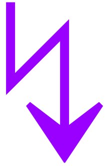
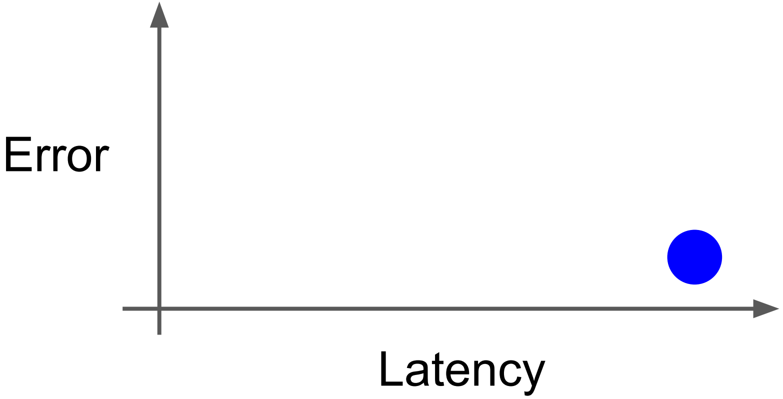
Candidates





$$\sqrt{\frac{1 + \left(e^{x}\right)^{1.5} \cdot \left(e^{x}\right)^{1.5}}{1 + \left(e^{x} + \left(e^{x}\right)^{1.5}\right) \cdot \left(\sqrt{e^{x}} - 1\right)}}$$



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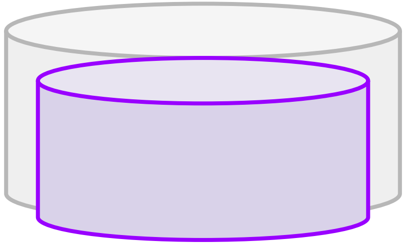

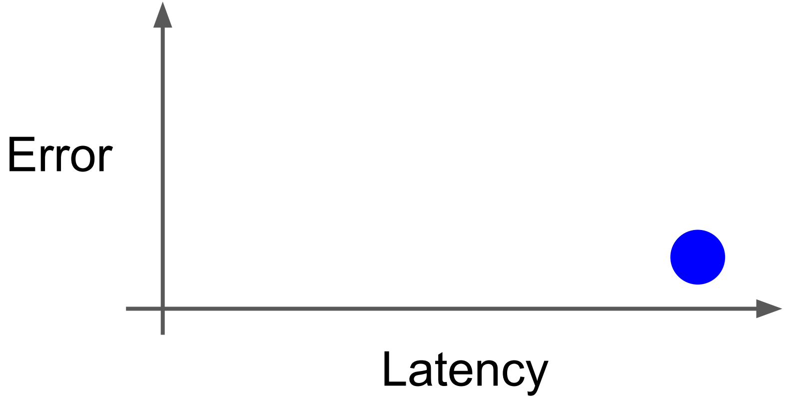

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Pherbie Regimes Example : Iter 2 / 5

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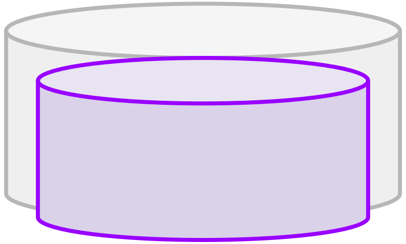
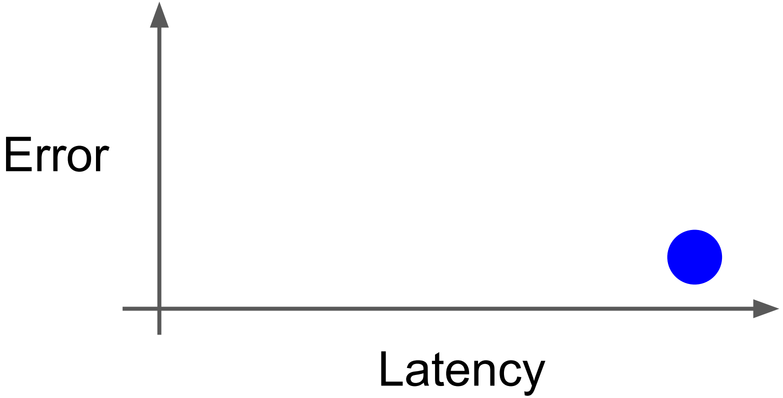
Candidates





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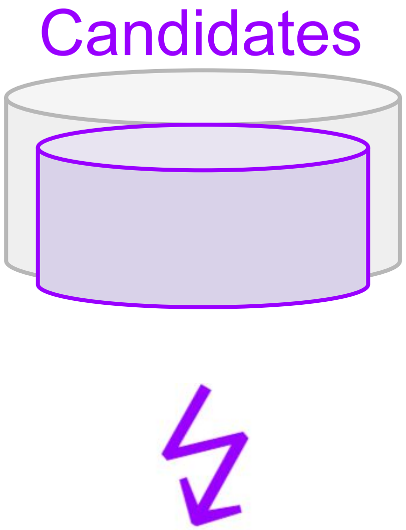
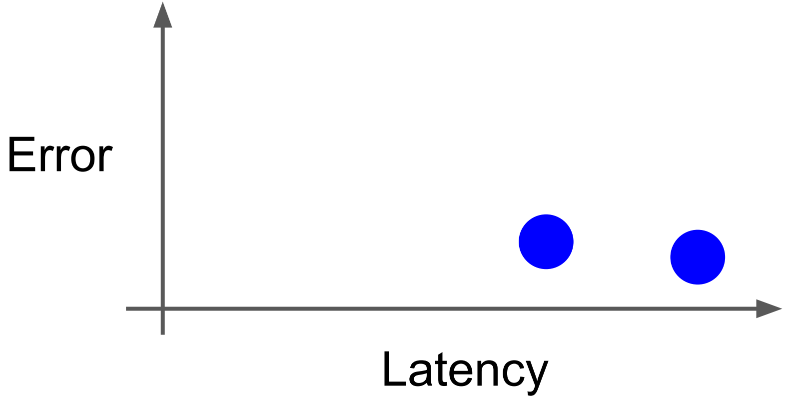


$$\sqrt{1 + e^{x}}$$



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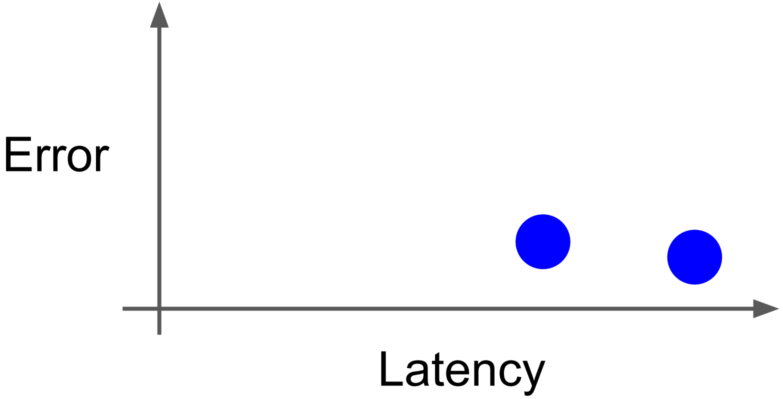
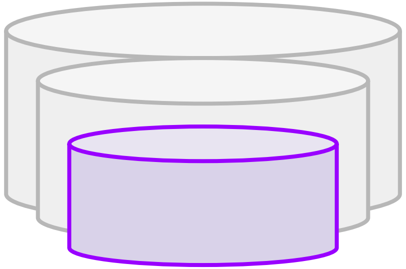


Candidates



$$\sqrt{1 + e^{x}}$$

Pherbie Regimes Example : Iter 3 / 5

$$\sqrt{\frac{e^{2x} - 1}{e^{x} - 1}}$$

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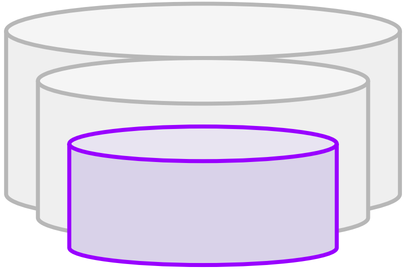
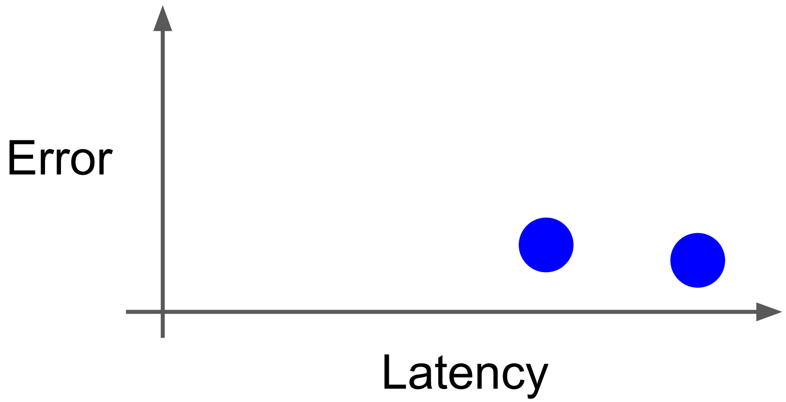
Candidates





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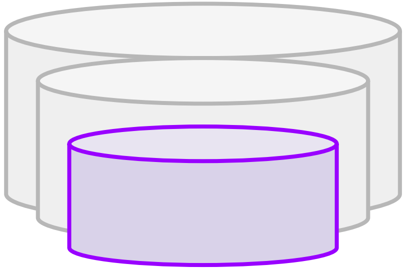

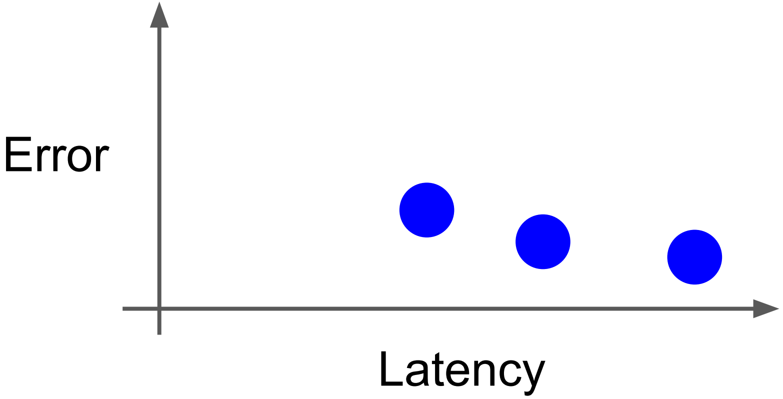

$$\begin{array}{l}\textbf{if } x \leq -0.10591501462198885 : \\ \quad \langle (\sqrt{1 + e^{x}})_{(\text{float } 5\ 16)} \rangle_{\text{binary64}} \\ \textbf{else} : \\ \quad \sqrt{2 + (x + x \cdot (x \cdot 0.5))}\end{array}$$

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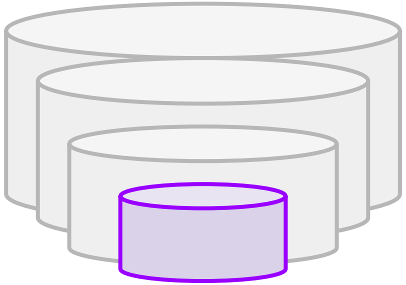
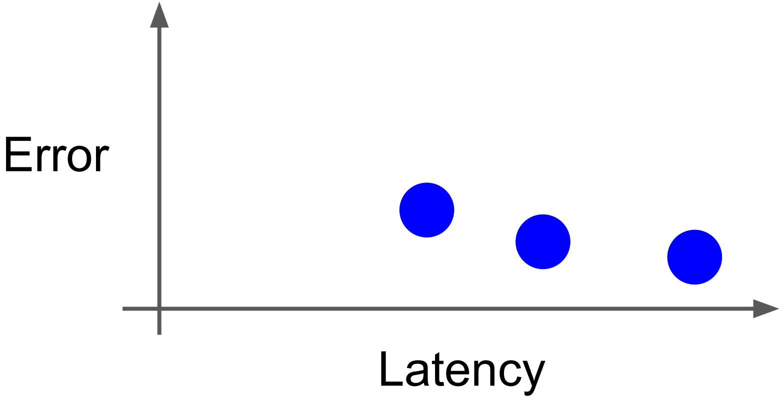
Candidates





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Pherbie Regimes Example : Iter 4 / 5

$$\sqrt{\frac{e^{2x} - 1}{e^x - 1}}$$

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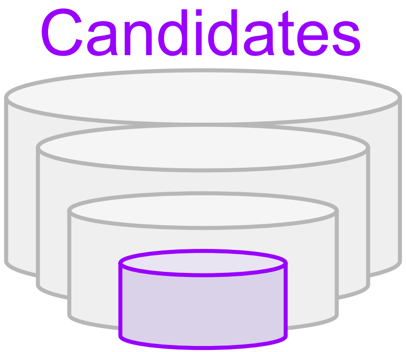

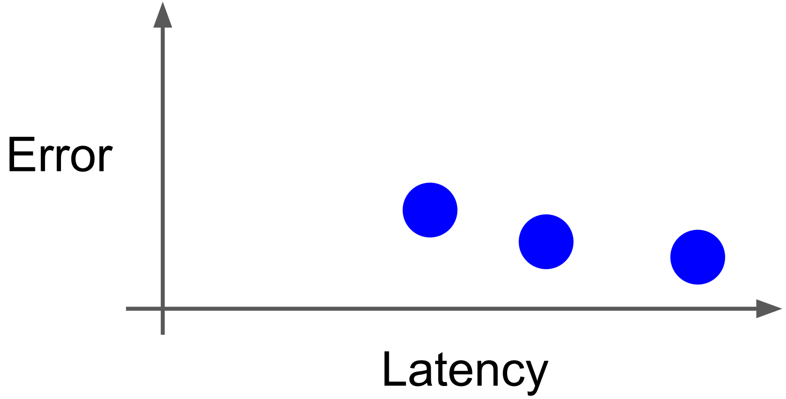

Pherbie Regimes Example : Iter 4 / 5

$$\sqrt{\frac{e^{2x} - 1}{e^{x} - 1}}$$

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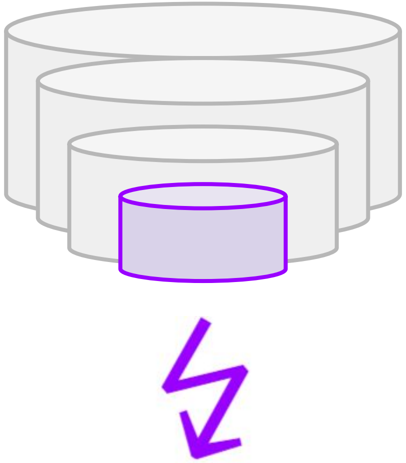
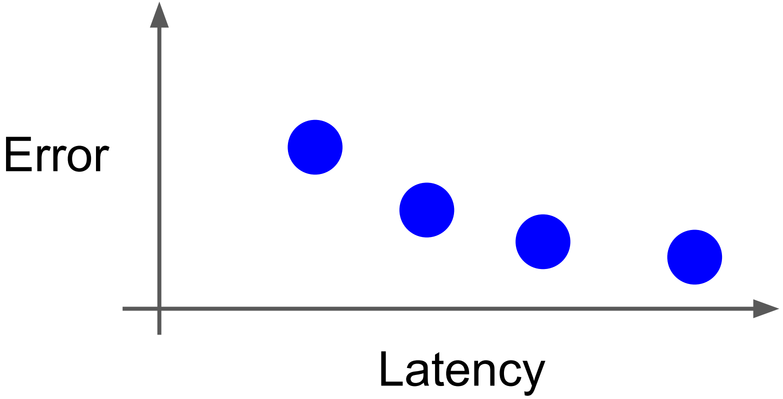
```

Candidates



$$\sqrt{2 + (x + x \cdot (x \cdot 0.5))}$$



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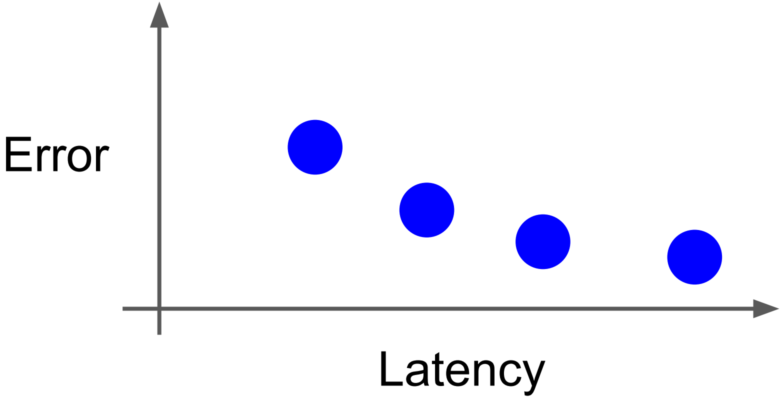

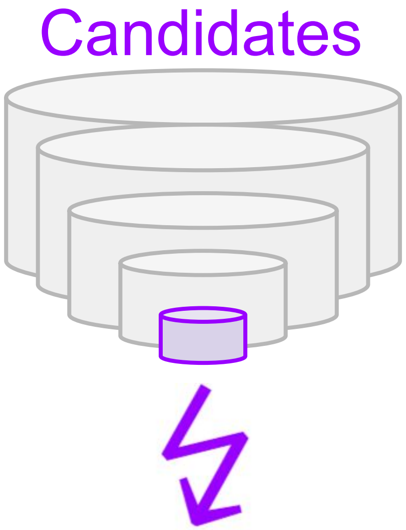

$$\sqrt{2 + (x + x \cdot (x \cdot 0.5))}$$

Pherbie Regimes Example : Iter 5 / 5

$$\sqrt{\frac{e^{2x} - 1}{e^{x} - 1}}$$

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```
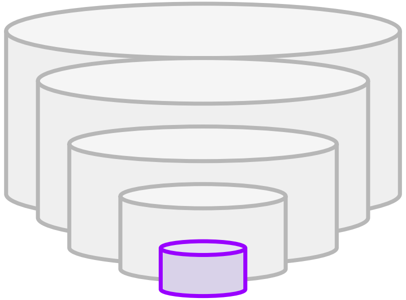
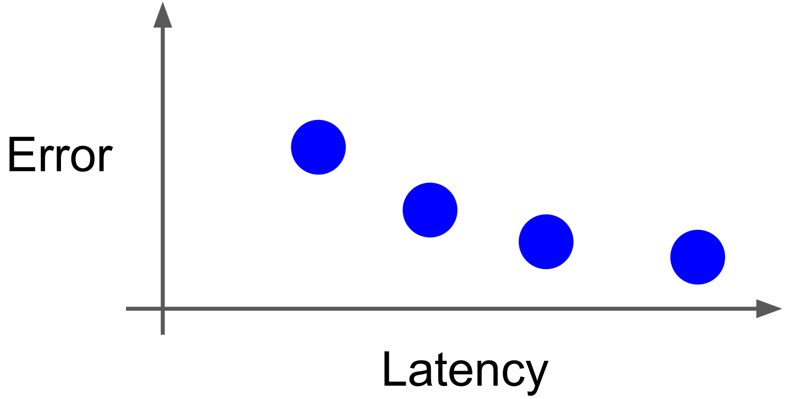

Candidates





Pherbie Regimes Example : Iter 5 / 5

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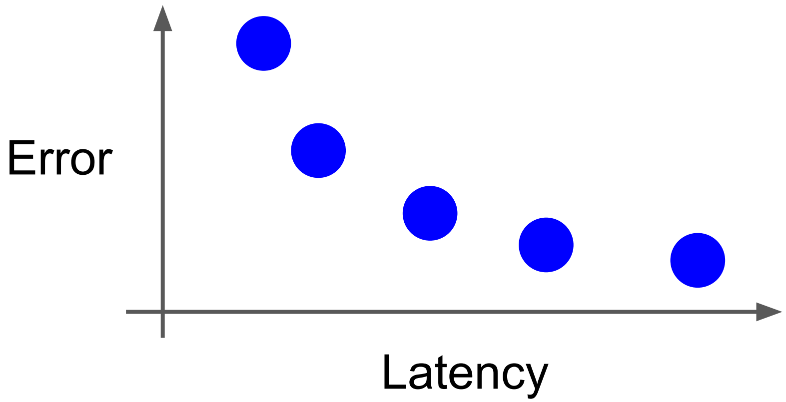
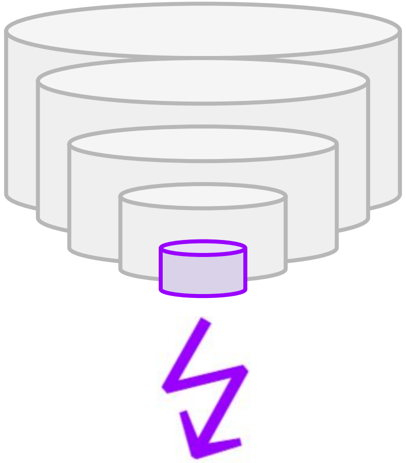


Candidates



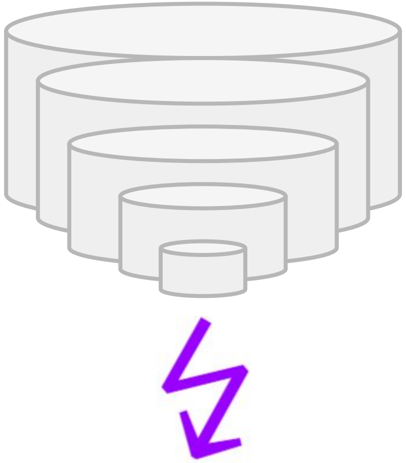
$$\sqrt{2}$$

Pherbie Regimes Example : Iter 5 / 5

$$\sqrt{\frac{e^{2x}-1}{e^{x}-1}}$$

```
while 0 < |Candidates| :
    p = ExtractMinError(Candidates)
    Candidates.removeAboveCost(p)
```

Candidates



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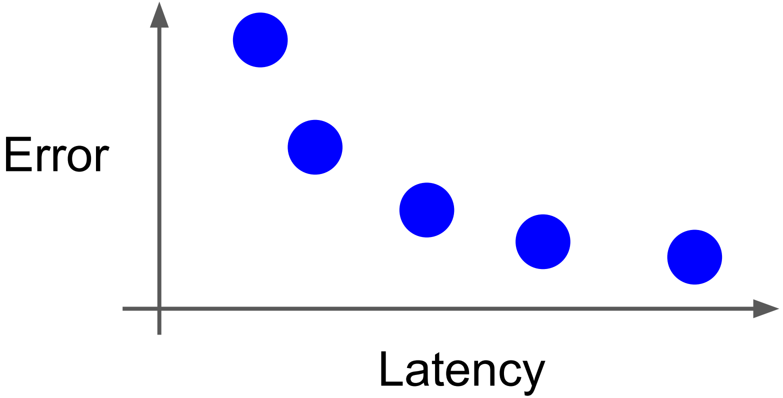

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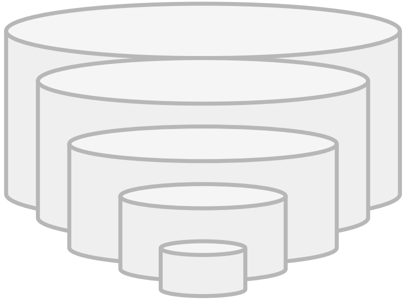
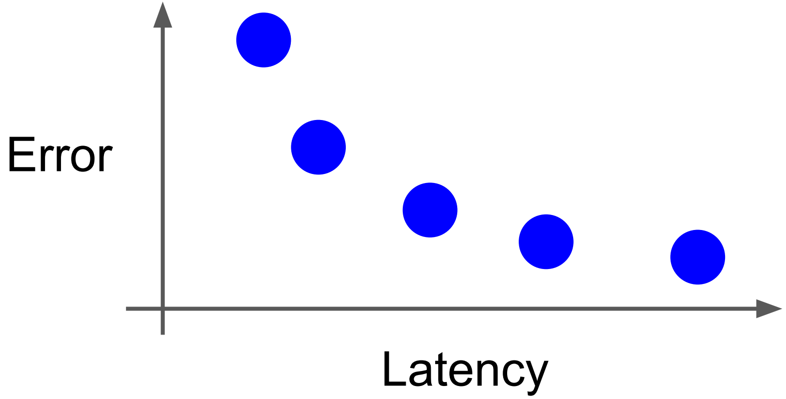
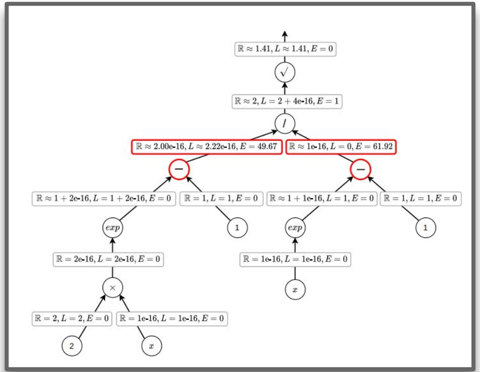




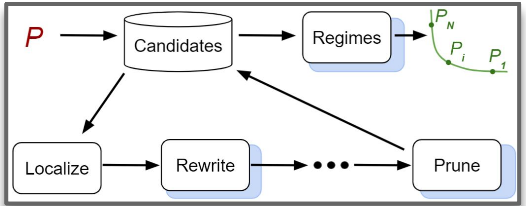
Outline

✓ **Herbie**: Improving Accuracy via Rewriting



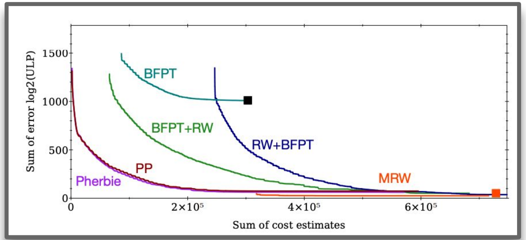
- Key Insight: local error guides rewriting

✓ **Pherbie**: Extending Herbie with Precision Tuning



- Key Insight: local error also guides precision tuning!

❏ **Evaluation**: Applying Pherbie to Classics + Graphics



- Key Insight: Finer-grained interleaving → better optimization!

Evaluation: Benchmark Suites

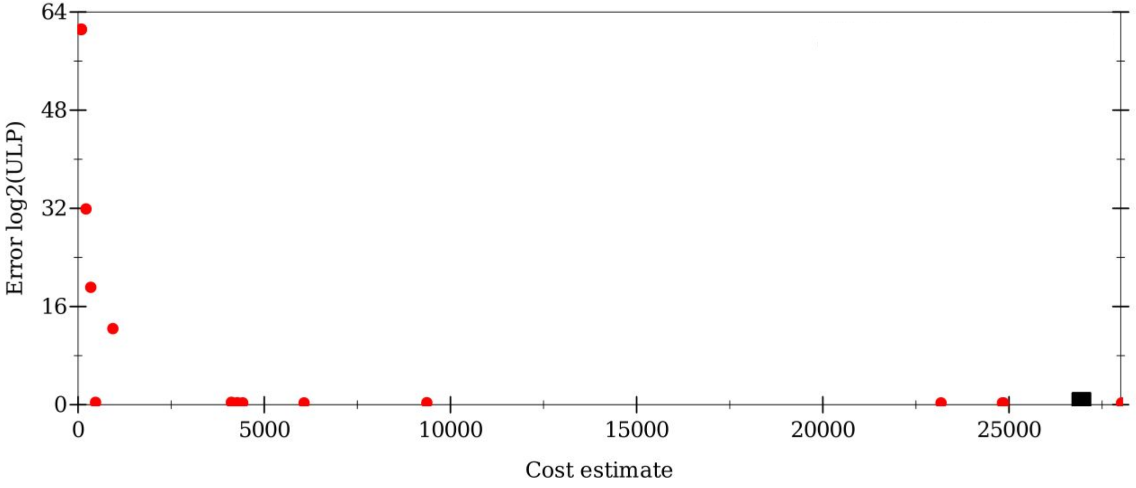
- NMSE - *Numerical Methods for Scientists and Engineers* (Hamming, 1986)
  - Standard textbook on numerical analysis
- PBRT - *Physically Based Rendering* (Pharr et. al, 2016)
  - Open-source textbook describing rendering photorealistic scenes

Evaluation

Pherbie produces Pareto-optimal implementations

Curve Intersection (PBRT)

$$\left(\sin\left(\left(1-u\right)\cdot normAngle\right)\cdot\frac{1}{\sin normAngle}\right)\cdot n0_i+\left(\sin\left(u\cdot normAngle\right)\cdot\frac{1}{\sin normAngle}\right)\cdot n1_i$$



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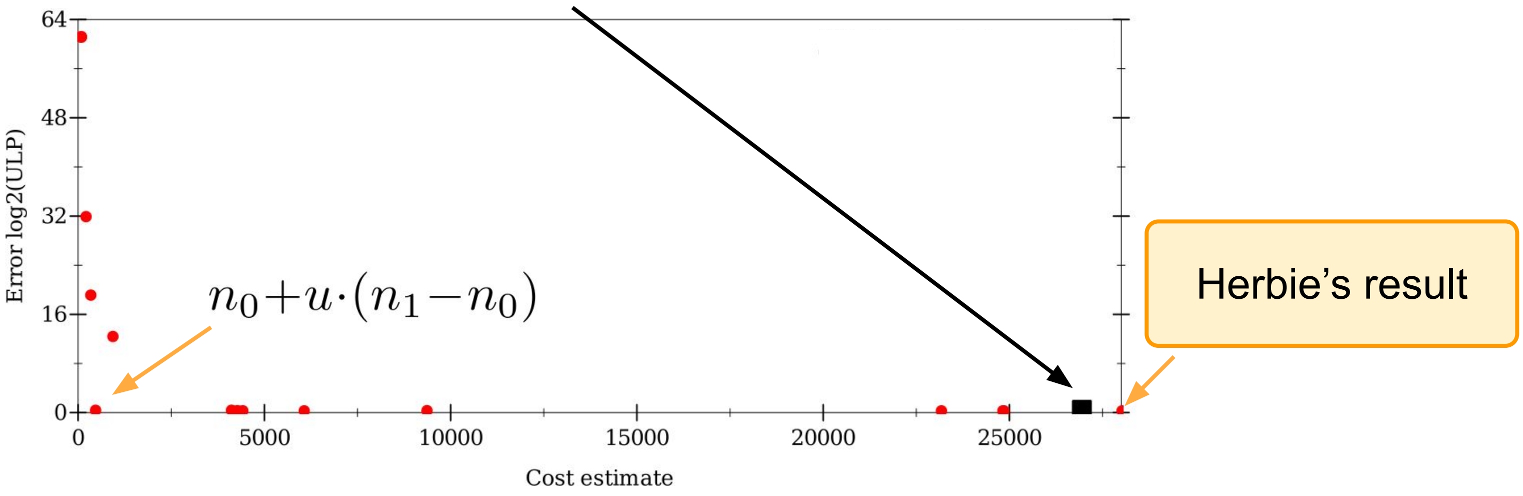

Evaluation

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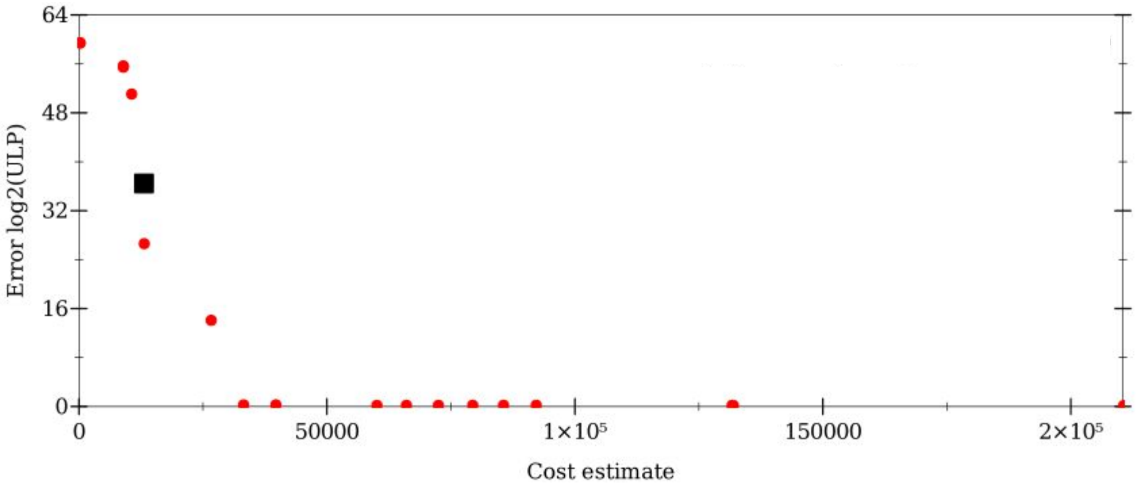
Nearby Tangent Difference (NMSE)

$$\tan(x + \epsilon) - \tan x$$



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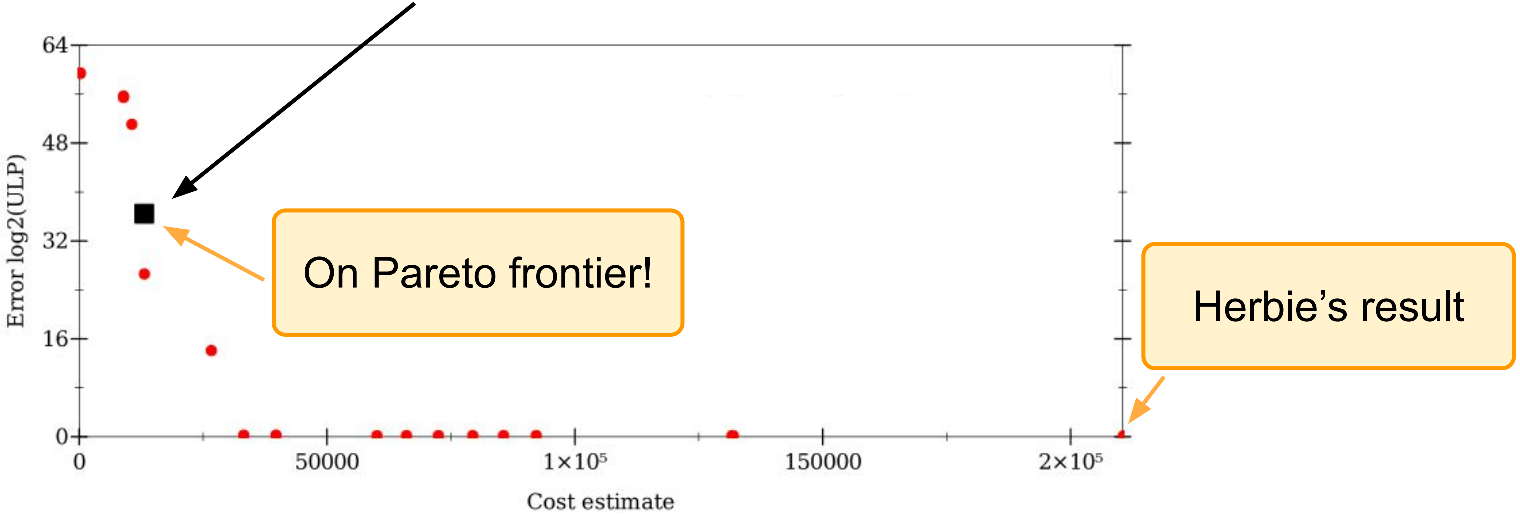
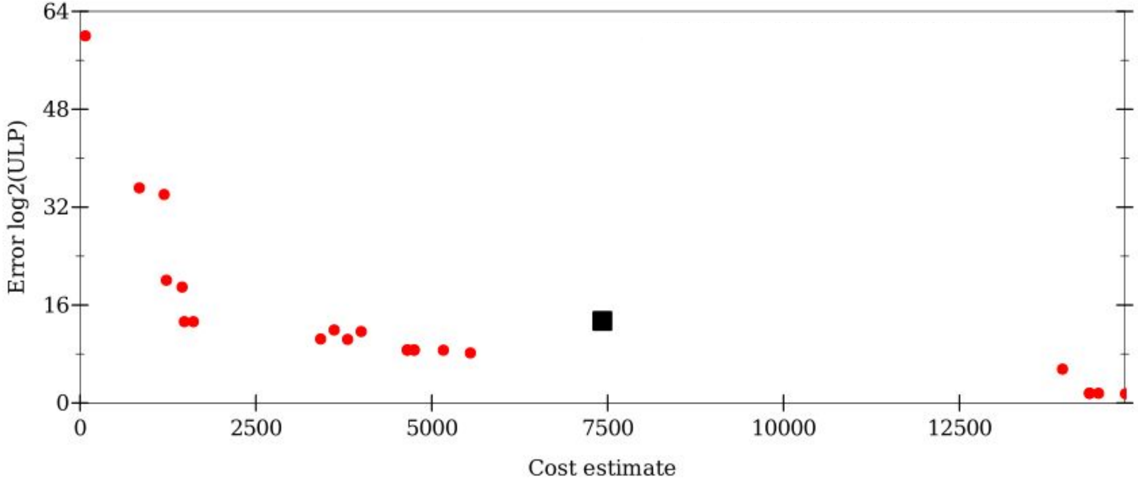


Evaluation

Pherbie produces Pareto-optimal implementations

Beckmann Distribution Sampling (PBRT)

$$\frac{-\log(1+u)}{c2p/(\alpha_x \cdot \alpha_x) + s2p/(\alpha_y \cdot \alpha_y)}$$



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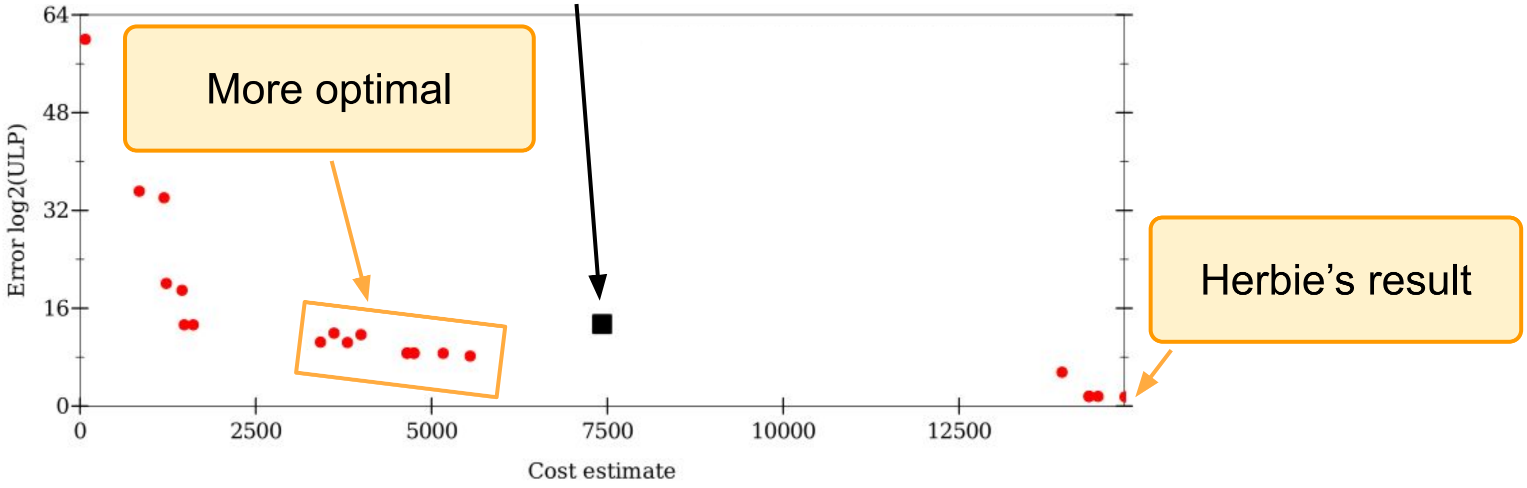


Evaluation

Finer interleavings ⇒ Better Pareto frontier

Comparing different methods of using rewriting and precision tuning:

Single Technique	Chaining Techniques	Interleaving Techniques
Herbie Herbie x100 (RW) Tuning-only (BFPT)	Rewrite-then-tune (RW+BFPT) Tune-then-rewrite (BFPT+RW)	Coarse-grained interleaving (PP) Fine-grained interleaving (Pherbie)

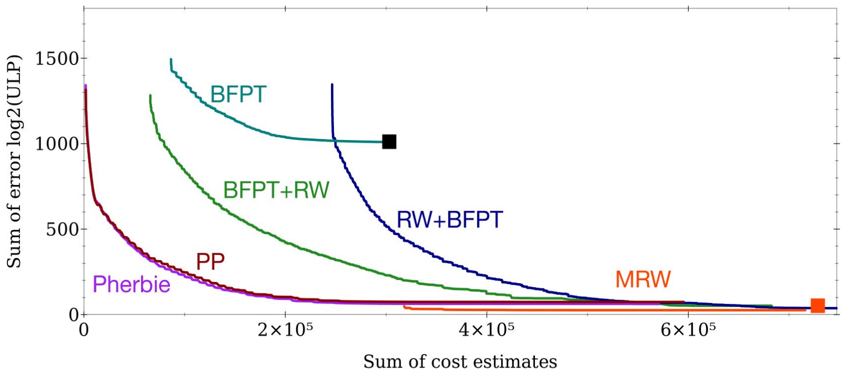
Evaluation

Finer interleavings ⇒ Better Pareto frontier

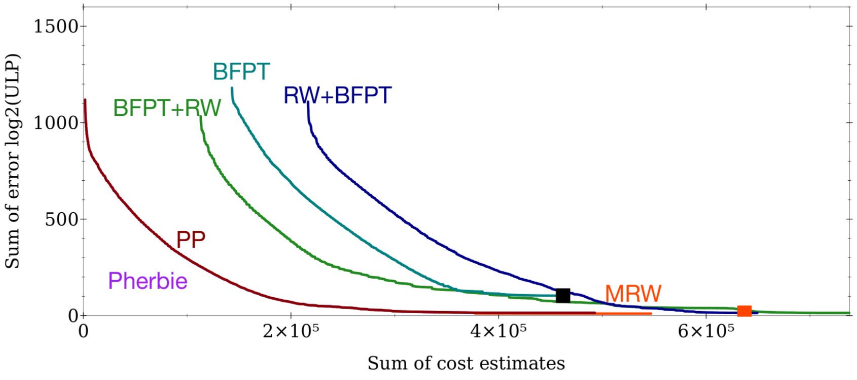
Method:

- For a given cumulative cost, what is the minimum cumulative error we can achieve by selecting one output expression from each benchmark?

NMSE



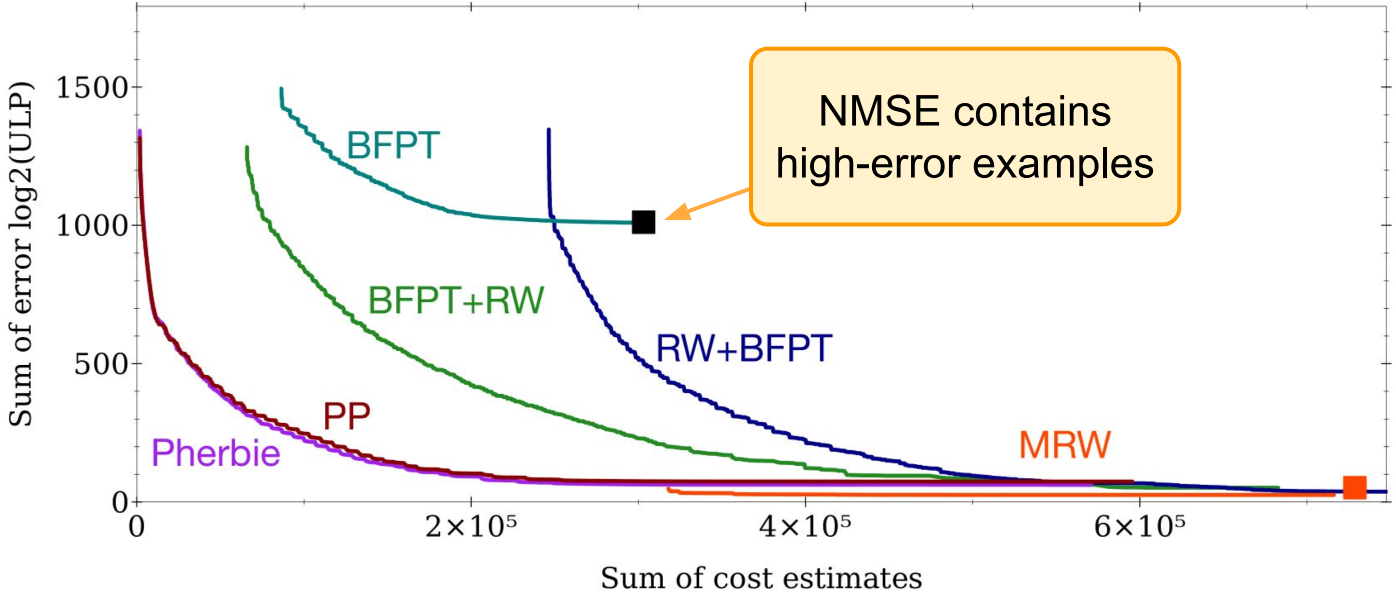
PBRT



Evaluation

Finer interleavings ⇒ Better Pareto frontier

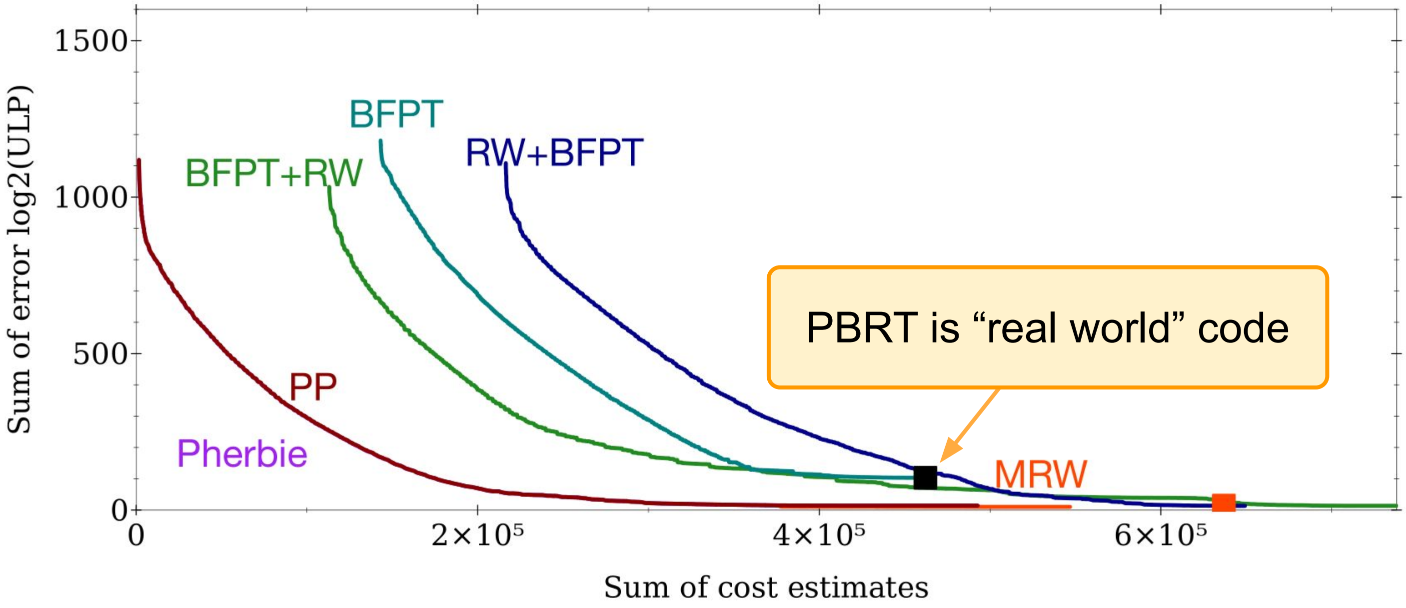
Suite: NMSE



Evaluation

Finer interleavings ⇒ Better Pareto frontier

Suite: PBRT



Outline

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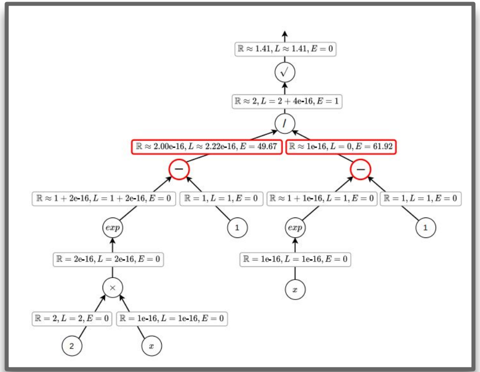

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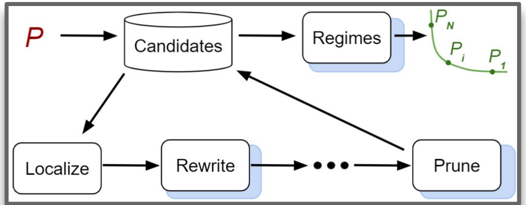

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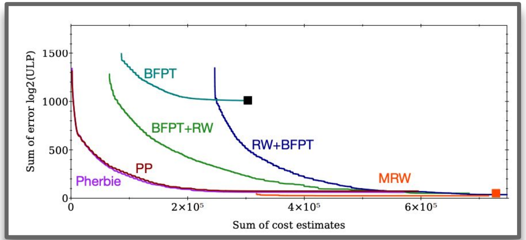
- Key Insight: Finer-grained interleaving → better optimization!



Related Work

- Scalable error analysis
  - [Gopalakrishnan et al. SC’20]
- Improving accuracy of imperative floating point programs
  - [Martel et al. AFM’17]
- Tunable precision of floating point programs
  - [Schkufza et al. PLDI ‘14]
- Sound compilation of real computations
  - [Darulova et al. POPL’14]
- Debugging and correct rounding of floating point programs
  - [Nagarakatte et al. POPL’21, PLDI’21]

Team and Acknowledgments

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UW

Pavel Panchekha
Univ. of Utah

Zachary Tatlock
UW

ada
Applications Driving Architectures

NSF

THANK YOU!

Pherbie: Precision Tuning + Rewriting

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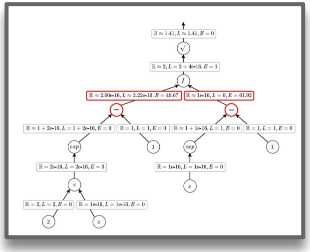

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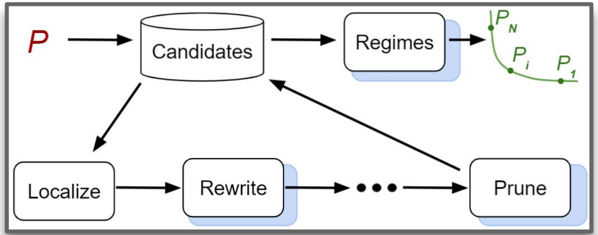

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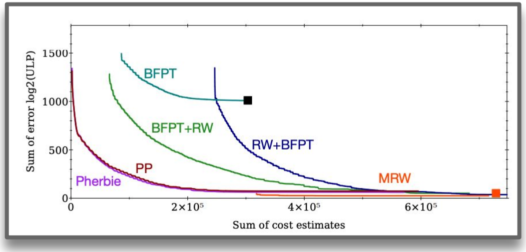

HERBIE

herbie.uwplse.org